Rayleigh-wave diffractions due to a void in the layered half space

Jianghai Xia, Kansas Geological Survey, The University of Kansas;*
Yixian Xu, State Key Laboratory of Geological Processes and Mineral Resources, China University of Geosciences;
Richard D. Miller, Kansas Geological Survey, The University of Kansas; and
Jonathan E. Nyquist, Geology Department, Temple University

Summary

Void detection is challenging due to the complexity of near-surface materials and the limited resolution of geophysical methods. Although multichannel, high-frequency, surface-wave techniques can provide reliable shear (S)-wave velocities in different geological settings, they are not suitable for detecting voids directly based on anomalies of the S-wave velocity because of limitations on the resolution of S-wave velocity profiles inverted from surface-wave phase velocities. Xia et al. (2006a) derived a Rayleigh-wave diffraction traveltime equation due to a void in the homogeneous half space. Encouraging results of directly detecting a void from Rayleigh-wave diffractions were presented (Xia et al., 2006a). In this paper we used four two-dimensional square voids in the layered half space to demonstrate the feasibility of detecting a void with Rayleigh-wave diffractions. Rayleigh-wave diffractions were recognizable for all these models after removing direct surface waves by F-K filtering. We evaluate the feasibility of applying the Rayleigh-wave diffraction traveltime equation to a void in the layered earth model. The phase velocity of diffracted Rayleigh waves is predominately determined by surrounding materials of a void. The modeling results demonstrate that the Rayleigh-wave diffraction traveltime equation due to a void in the homogeneous half space can be applied to the case of a void in the layered half space. In practice, only two diffraction times are necessary to define the depth to the top of a void and the average velocity of diffracted Rayleigh waves.

Introduction

Elastic properties of near-surface materials and their effects on seismic wave propagation are of fundamental interest in ground-water, engineering, and environmental studies. Shear (S)-wave velocities can be derived from inverting dispersive phase velocities of the surface (Rayleigh and/or Love) waves (e.g., Dorman and Ewing, 1962). Multichannel Analysis of Surface Waves—MASW (Park et al., 1999; Xia et al., 1999)—possesses the advantage of easily recognizing surface waves, effectively eliminating body-wave energy, and accurately defining dispersion energy. Errors associated with S-wave velocities obtained by the MASW method are 15% or less and random after comparison with borehole direct measurements (Xia et al., 2000, 2002a, and 2002b). If higher-mode data are available, the accuracy of an inverted S-wave velocity can be significantly improved (Xia et al., 2003; Beaty et al., 2002; Beaty and Schmitt, 2003). More publications have appeared on utilizing surface waves in defining S-wave velocities such as a pitfall in S-wave refraction (Xia et al., 2002b), acquisition parameters (Xia et al., 2006b; Xu et al., in press), attenuation properties (Xia et al., 2002c), delineation of a collapse feature in an extremely noisy environment (Xia et al., 2004), seismic zonation (Yilmaz et al., 2006); and S-wave velocities of a non-layered earth model (Xia et al., in press).

Although multichannel, high-frequency, surface-wave techniques can provide reliable S-wave velocities in different geological settings, they are not suitable for detecting voids directly based on anomalies of the S-wave velocity because of limitations on the resolution of S-wave velocity profiles inverted from surface-wave phase velocities. Several researchers have published results on void detection with surface waves. Inversion of scattered surface waves has been discussed by Riyanti et al. (2005). They concluded that scattered surface waves can be used for near-surface characterization using inversion methods. Analysis attenuation of Rayleigh waves (AARW) for detecting voids has been presented (Nasseri-Moghaddam et al., 2005). The numerical results show the promise of AARW for detecting voids in the real world. Gelis et al. (2005) have reported numerical modeling results. A cavity with a rectangular cross section diffracts more energy than a cavity with a circular cross section. They also found that a low-velocity zone around and above a cavity might increase Rayleigh-wave attenuation and could possibly mask the cavity signature.

Xia et al. (2006a) derived a Rayleigh-wave diffraction traveltime equation due to a void in the homogeneous half space. Encouraging results showing direct detection of a void from Rayleigh-wave diffractions were presented. In this paper we used four two-dimensional (2-D) square voids in the layered half space to demonstrate the feasibility of detecting a void with Rayleigh-wave diffractions. Rayleigh-wave diffractions were recognizable for all these models after F-K filtering was applied to remove the direct surface waves. We evaluate the feasibility of applying the Rayleigh-wave diffraction traveltime equation to a void in the layered earth model. The phase velocity of diffracted Rayleigh waves is predominately determined by materials surrounding the void. The modeling results demonstrate that the Rayleigh-wave diffraction traveltime equation due to a void in the homogeneous half space can be applied to the case of a void in the layered half space. In practice, only two diffraction times are necessary to define the depth to the top of a void and the average Rayleigh-wave velocity that generates the diffraction curve.

The diffraction equation of surface waves

Xia et al. (2006a) derived the diffraction equation of surface waves due to a void in a homogeneous half space. Suppose surface waves with a dominant phase velocity *v* travel along the ground surface (direct surface waves, segment 1 in Figure 1); then a diffraction of surface waves with the same phase velocity due to a void can occur at the top left corner of the void (diffracted surface waves, segment 2 in Figure 1). The traveltime equation for the diffraction is

$$t_x = \frac{1}{v}\left[d + \left(x^2 + h^2\right)^{1/2}\right], \tag{1}$$

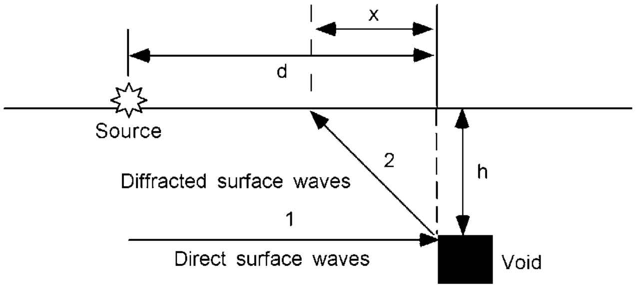

Figure 1. Geometry of surface-wave diffractions.

where t_x is the diffraction arriving time at the offset of *x* (the horizontal distance between the apex of the hyperbola corresponding to the edge of the void and a receiver), *v* is a phase velocity of the diffraction, *d* is the horizontal distance from the source to the apex of the hyperbola, and *h* is the depth to the top of the void (the diffraction point). The phase velocity *v* and the depth to the top of a void can be determined by two traveltimes extracted from the diffraction (Xia et al., 2006a). In practice, when $x >> h$, $t_x = (d + x)/v$ and *v* can be determined by a linear fit to the diffracted waves.

Modeling results

We placed a 2-D 2 m × 2 m square void in a layered earth model at a depth to the top of the void of 4 m (Figure 2). With a cell size of 1 m × 1 m, a 200 m × 200 m subspace was divided into 200 × 200 nodes with 30 nodes in the transition zones along the left and right sides of the model and 30 nodes in the transition zone along the bottom of the model. Synthetic seismographs simulated the following field layout. A source and geophones were on the ground surface. A source was at 70 m in the x-coordinate. Sixty vertical component geophones were located in the middle of the subspace from 71 to 130 m in the x-coordinate with an interval of 1 m. The void was at the center of the geophone spread. We modeled four models: Model 1—a low velocity layer on the top of the half space and the void in the half space (Figure 2a); Model 2—a three layer model with the void in the middle layer (Figure 2b); Model 3—a two layer model with the void in the top layer (Figure 2c); and Model 4—the same as the case 3 but with P- and S-wave velocities of the half space of 2000 m/s and 400 m/s, respectively, higher than Model 3. We used an algorithm developed by Xu et al. (2005) to generate synthetic seismographs. To avoid numerical difficulties, we used P- and S- wave velocities of the void at 340 m/s and 17 m/s, respectively. The density of the void was 10 kg/m^3.

Figure 3 shows a synthetic shot gather for void Model 1 with a source to diffraction point distance (*d*) of 29 m. Diffractions (Figure 3a) can be identified. Notice that the phase of diffractions possesses a 180-degree shift. Diffractions were relatively strong (Figure 3b) after removing linear events mainly direct surface waves with F-K filtering. Based on their frequency and velocity, we are certain that the diffractions are surface-wave diffractions due to the void at a depth of 4 m. We used Equation (1) with the phase velocity of 180 m/s and t_0 = 0.186 ms to model a traveltime curve (a solid hyperbolic line in Figure 3b). The traveltime curve matches the diffractions perfectly, which provides a depth to the top of the void of 4.5 m. The diffracted phase velocity of 180 ms is very close to the phase velocity of the half space of ~184 m/s (Sheriff and Geldart, 1985, p. 49). In this case, the lower velocity of the top layer has little effect on the diffracted waves.

Figure 4 shows F-K filtered results for the three other models. The diffracted waves possess a phase velocity of 190 m/s when the void is in the second layer of the three-layer model (Figure 4a). The small increase in diffracted velocity is due to the higher velocity of the half space. When removing the low velocity layer on the top of the Model 2, we obtain Model 3. The diffracted waves in this case possess a phase velocity of 200 m/s (Figure 4b). We know the phase velocity of the half space is around 276 m/s. We replace the half space in Model 3 with 33% higher velocities (Vp = 2000 m/s and Vs = 400 m/s) to obtain Model 4 to find how severe the effects are on the diffracted waves with changes in the half-space velocity. The diffracted waves in this case possess a phase velocity of 220 m/s (Figure 4c). We know the phase velocity of the half space is ~368 m/s. After defining diffracted phase velocity, a hyperbolic fitting to diffracted waves can provide a good estimate of t_0 or *h* (Figure 4).

Discussion and Conclusions

We studied feasibility of detecting a void with surface-wave diffractions in a layered earth model by 2-D surface-wave modeling. In most cases, the diffracted surface waves are masked by the direct surface waves so F-K filtering is necessary to remove the direct surface waves before analyzing diffracted waves. The modeling results demonstrated that the phase velocity of the diffracted surface waves is mainly determined by its surrounding materials. The traveltime equation of surface-wave diffractions due to a void in a homogeneous half space (Xia et al., 2006a) could be directly applied to a void in a layered model. Even though our studies were restricted to a 2-D model, and do not consider attenuation of surface waves, the results of this paper are directly applicable to the detection of 2-D structures such as tunnels. Because surface-wave diffractions are relatively weak, especially in a high attenuation medium, a real challenge in detecting a deeper void is to generate sufficient surface-wave energy with a long wavelength and a high signal-to-noise ratio.

Acknowledgments

The authors thank Mary Brohammer of the Kansas Geological Survey for editing the manuscript.

References

Beaty, K.S., and Schmitt, D.R., 2003, Repeatability of multimode Rayleigh-wave dispersion studies, Geophysics, 68(3), 782-790.

Beaty, K.S., Schmitt, D.R., and Sacchi, M., 2002, Simulated annealing inversion of multimode Rayleigh wave dispersion curves for geological structure: Geophys. J. Int., 151, 622–631.

Dorman, J., and Ewing, M., 1962, Numerical inversion of seismic surface wave dispersion data and Crust-Mantle structure in the New York-Pennsylvania area: J. Geophys. Res., 67, 5227-5241.

Gelis, C., Leparoux, D., Virieux, J., Bitri, A., Operto, S., and Grandjean, G., 2005, Numerical modeling of surface waves over shallow cavities: Journal of Environmental and Engineering Geophysics, 10(2), 111-121.

Nasseri-Moghaddam, A., Cascante, G., and Hutchinson, J., 2005, A new quantitative procedure to determine the location and embedment depth of a void using surface waves: Journal of Environmental and Engineering Geophysics, 10(1), 51-64.

Park, C.B., Miller, R.D., and Xia, J., 1999, Multi-channel analysis of surface waves: Geophysics, 64(3), 800-808.

Riyanti, C.D., Campman, X., and Herman, G., 2005, Inversion of scattered surface waves for characterizing shallow heterogeneities: Technical Program with Biographies, SEG, 74th Annual Meeting, Houston, TX, 1053-1056.

Sheriff, R.E., and Geldart, L.P., 1985, Exploration seismology (volume 1): History, theory, and data acquisition: Cambridge University Press, New York.

Xia, J., Miller, R.D., and Park, C.B., 1999, Estimation of near-surface shear-wave velocity by inversion of Rayleigh wave: Geophysics, 64(3), 691-700.

Xia, J., Miller, R.D., Park, C.B., Hunter, J.A., and Harris, J.B., 2000, Comparing shear-wave velocity profiles from MASW with borehole measurements in unconsolidated sediments, Fraser River Delta, B.C., Canada: Journal of Environmental and Engineering Geophysics, 5(3), 1-13.

Xia, J., Miller, R.D., Park, C.B., Hunter, J.A., Harris, J.B., and Ivanov, J., 2002a, Comparing shear-wave velocity profiles from multichannel analysis of surface wave with borehole measurements: Soil Dynamics and Earthquake Engineering, 22(3), 181-190.

Xia, J., Miller, R.D., Park, C.B., Wightman, E., and Nigbor, R., 2002b, A pitfall in shallow shear-wave refraction surveying: Journal of Applied Geophysics, 51(1), 1-9.

Xia, J., Miller, R.D., Park, C.B., and Tian, G., 2002c, Determining Q of near-surface materials from Rayleigh waves: Journal of Applied Geophysics, 51(2-4), 121-129.

Xia, J., Miller, R.D., Park, C.B., and Tian, G., 2003, Inversion of high frequency surface waves with fundamental and higher modes: Journal of Applied Geophysics, 52(1), 45-57.

Xia, J., Chen, C., Li, P.H., and Lewis, M.J., 2004, Delineation of a collapse feature in a noisy environment using a multichannel surface wave technique: Geotechnique, 54(1), 17-27.

Xia, J., Nyquist, J.E., Xu, Y., and Roth, M.J.S, 2006a, Feasibility of detecting voids with Rayleigh-wave diffraction: Proceedings of the Symposium on the Application of Geophysics to Engineering and Environmental Problems (SAGEEP) Annual Meeting of the Environmental and Engineering Geophysical Society (EEGS), April 2-6, 2006, Seattle, WA, 1168-1180.

Xia, J., Xu, Y., Chen, C., Kaufmann, R.D., and Luo, Y., 2006b, Simple equations guide high-frequency surface-wave investigation techniques: Soil Dynamics and Earthquake Engineering. 26(5), 395-403.

Xia, J., Xu, Y., Miller, R.D., and Chen, C., in press, Estimation of elastic moduli in a compressible Gibson half-space by inverting Rayleigh wave phase velocity: Surveys in Geophysics.

Xu, Y., Xia, J., and Miller, R.D., 2005, Finite-difference modeling of high-frequency Rayleigh waves: Technical Program with Biographies, SEG, 75th Annual Meeting, Houston, TX, 1057-1060.

Xu, Y., Xia, J., and Miller, R.D., in press, Quantitative estimation of minimum offset for multichannel surface-wave survey with actively exciting source: Journal of Applied Geophysics.

Yilmaz, O., Eser, M., and Berilgen, M., 2006, A case study of seismic zonation in municipal areas: The Leading Edge, 25(3), 319-330.

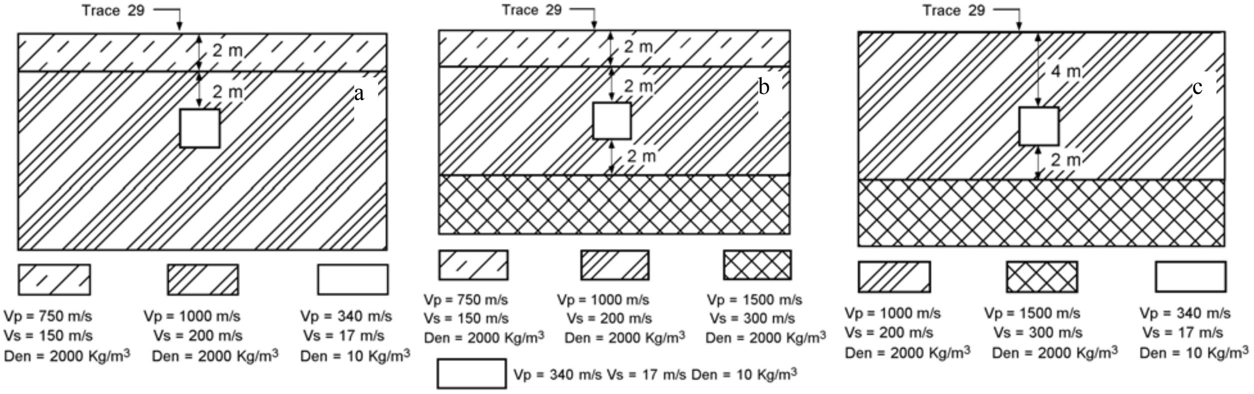

Figure 2. A two-dimensional 2 m × 2 m square void in layered earth models at a depth to its top of 4 m. The apex of diffractions is located at trace 29 of synthetic shot gathers.

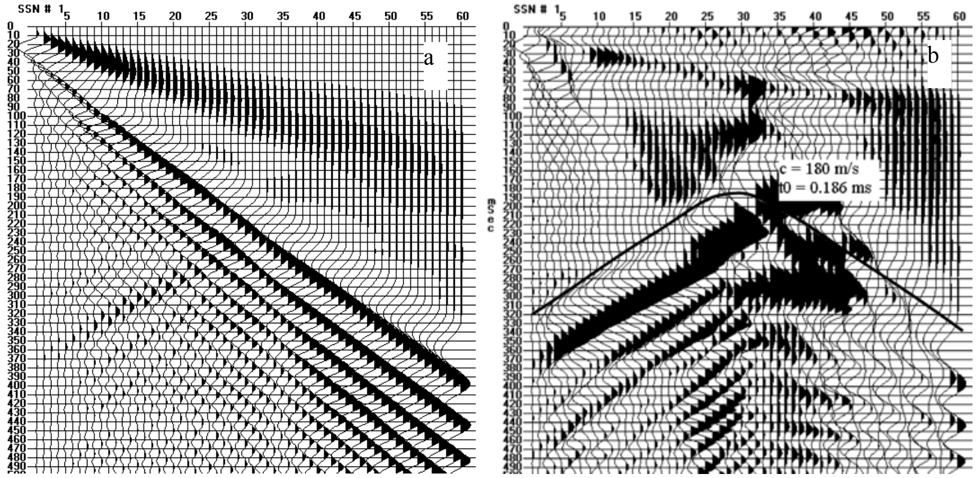

Figure 3. (a) Synthetic shot gather due to Model 1 (Fig. 2a). (b) A F-K filter was applied to synthetic shot gather (a) to remove the direct surface waves. Diffracted surface waves were obvious after direct surface waves were removed. A solid hyperbolic line with phase velocity of 180 m/s and t_0 ($x = 0$ in Equation 1) of 0.186 ms was superposed on F-K filtered data (b).

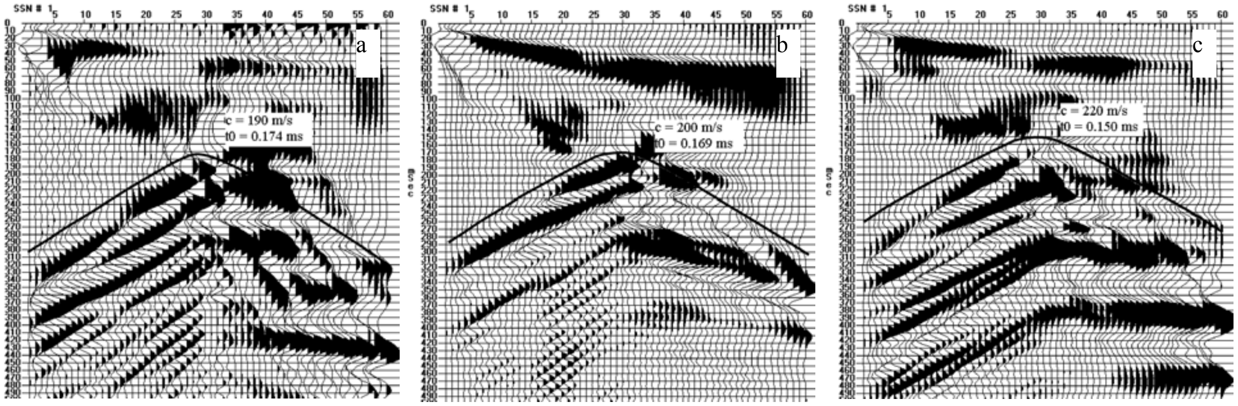

Figure 4. F-K filtered results of three other models and solid hyperbolic lines determined with phase velocity c and t_0 ($x = 0$ in Equation 1). (a) F-K filtered shot gather of Model 2 (Figure 2b). (b) F-K filtered shot gather of Model 3 (Figure 2c). (c) F-K filtered shot gather of Model 4 (see text for the model parameters). Based on c and t_0, depths to the top of the void for Models 2, 3, and 4 are 4.1 m, 4.8 m, and 4.0 m, respectively.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2006 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Beaty, K. S., and D. R. Schmitt, 2003, Repeatability of multimode Rayleigh-wave dispersion studies: Geophysics, **68**, 782–790.

Beaty, K. S., D. R. Schmitt, and M. Sacchi, 2002, Simulated annealing inversion of multimode Rayleigh wave dispersion curves for geological structure: Geophysical Journal International, **151**, 622–631.

Dorman, J., and M. Ewing, 1962, Numerical inversion of seismic surface wave dispersion data and Crust-Mantle structure in the New York-Pennsylvania area: Journal of Geophysical Research, **67**, 5227–5241.

Gelis, C., D. Leparoux, J. Virieux, A. Bitri, S. Operto, and G. Grandjean, 2005, Numerical modeling of surface waves over shallow cavities: Journal of Environmental and Engineering Geophysics, **10**, 111–121.

Nasseri-Moghaddam, A., G. Cascante, and J. Hutchinson, 2005, A new quantitative procedure to determine the location and embedment depth of a void using surface waves: Journal of Environmental and Engineering Geophysics, **10**, 51-64.

Park, C. B., R. D. Miller, and J. Xia, 1999, Multi-channel analysis of surface waves: Geophysics, **64,** 800–808.

Riyanti, C. D., X. Campman, and G. Herman, 2005, Inversion of scattered surface waves for characterizing shallow heterogeneities: 74th Annual International Meeting, SEG, Expanded Abstracts, 1053–1056.

Sheriff, R. E., and L. P. Geldart, 1985, Exploration seismology (volume 1): History, theory, and data acquisition: Cambridge Univ. Press.

Xia, J., C. Chen, P. H. Li, and M. J. Lewis, 2004, Delineation of a collapse feature in a noisy environment using a multichannel surface wave technique: Geotechnique, **54**, 17–27.

Xia, J., R. D. Miller, and C. B. Park, 1999, Estimation of near-surface shear-wave velocity by inversion of Rayleigh wave: Geophysics, **64**, 691–700.

Xia, J., R. D. Miller, C. B. Park, J. A. Hunter, and J. B. Harris, 2000, Comparing shear-wave velocity profiles from MASW with borehole measurements in unconsolidated sediments, Fraser River Delta, B.C.: Canada: Journal of Environmental and Engineering Geophysics, **5**, 1–13.

Xia, J., R. D. Miller, C. B. Park, J. A. Hunter, J. B. Harris, and J. Ivanov, 2002a, Comparing shear-wave velocity profiles from multichannel analysis of surface wave with borehole measurements: Soil Dynamics and Earthquake Engineering, **22**, 181–190.

Xia, J., R. D. Miller, C. B. Park, and G. Tian, 2002, Determining Q of near-surface materials from Rayleigh waves: Journal of Applied Geophysics, **51,** 121–129.

———, 2003, Inversion of high frequency surface waves with fundamental and higher modes: Journal of Applied Geophysics, **52,** 45–57.

Xia, J., R. D. Miller, C. B. Park, E. Wightman, and R. Nigbor, 2002b, A pitfall in shallow shear-wave refraction surveying: Journal of Applied Geophysics, **51,** 1–9.

Xia, J., J. E. Nyquist, Y. Xu, and M. J. S. Roth, 2006a, Feasibility of detecting voids with Rayleigh-wave diffraction: Proceedings of the SAGEEP Annual Meeting of the Environmental and Engineering Geophysical Society, 1168–1180.

Xu, Y., J. Xia, and R. D. Miller, 2005, Finite-difference modeling of high-frequency Rayleigh waves: 75th Annual International Meeting, SEG, Expanded Abstracts, 1057–1060.

———, 2006, Quantitative estimation of minimum offset for multichannel surface-wave survey with actively exciting source: Journal of Applied Geophysics, in press.

Xia, J., Y. Xu, C. Chen, R. D. Kaufmann, and Y. Luo, 2006b, Simple equations guide high-frequency surface-wave investigation techniques: Soil Dynamics and Earthquake Engineering, **26**, 395–403.

Xia, J., Y. Xu, R. D. Miller, and C. Chen, 2006, Estimation of elastic moduli in a compressible Gibson half-space by inverting Rayleigh wave phase velocity: Surveys in Geophysics.

Yilmaz, O., M. Eser, and M. Berilgen, 2006, A case study of seismic zonation in municipal areas: The Leading Edge, **25**, 319–330.