High-order correlative weighted stacking for seismic data in wavelet domain

Sanzong Zhang, Yixian Xu, China University of Geosciences, Wuhan, China; and*
Jianghai Xia, Kansas Geological Survey, The University of Kansas

Summary

Random noises in adjacent traces sometime exhibit coherence and correlation. The weighted stacking technique based on the conventional correlative function results in false events that are caused by the noises. Based on the theory of wavelets and high-order statistics, a high-order correlative weighted stacking technique is present in this paper. Its essence is to stack common midpoint gathers after the normal moveout correction by weights that are calculated through high-order correlative statistics in the wavelet domain. Synthetic examples demonstrate its advantages in improving the signal to noise ratio and compressing the correlative random noise.

Introduction

Horizontal stacking plays an important role in modern seismic data processing, due to its effectiveness in noise and multiplies suppressing in order to get a coarse imaging that approximates the underground structure and provide foundational information for subsequent inversion, migration, and image. Conventional stacking, however, adopts average stacking. Practice has proved that it could not enhance signal to noise (S/N) ratio to the maximum content and degrade resolution. Therefore, in the recent years some improved stacking methods came out subsequently. Robinson (1970) firstly introduced stacking according to weights that equal the second-order correlative coefficient of neighboring traces. This approach produced excellent result. On the basis of Robinson, Mao (2002) found another stacking by means of optimized weights. His approach is to compute weights by estimating amplitude spectrum and covariance of noises with a more practical signal model, and then optimize weights to maximize S/N ratio. Considering that the difference frequency spectrum has different S/N ratio, Wang (2003) suggested stacking with weights in different frequency components, where the weight factors are S/N ratio of seismic data computed quantitatively. The method mentioned above all use the second-order correlative coefficients of neighboring traces as weigh factors. According to the theory of high-order statistics, the second-order correlative analysis can inhibit uncorrelative random noises, but not for correlative random noises (Yung, 1997). A numerical example is given to prove this point in the following sections. From the requirement of resolution, compression of noises in the time domain only enhance S/N ratio of all and can not improve S/N ratio of some certain frequency components. As a result it is impossible to widen frequency spectrum and improve S/N ratio to the content, and it mat not be able to improve resolution. According to this point and taking into account the decomposition frequency characters of the wavelet transform, this paper presented a new kind of stacking that is executed in the wavelet domain so as to get a seismic profile with higher S/N ratio and resolution. Its basic theory is to perform the wavelet transform to seismic data, and then compute high-order correlative coefficients as weights for horizontal stacking. It is called a high-order correlative weighted stacking technique (HOCWS). Numerical examples show its effectiveness in compressing the correlative random noises.

Wavelet analysis and high-order correlative analysis

If f is in a square integral function space, L^2 can be expanded by its subspace W_j $L^2(R)=\bigoplus_{j\in Z} W_j$, we have $f(t)=\sum_{j=-\infty}^{+\infty} f_j(t)$, $f_j(t)=\sum_{k=-\infty}^{+\infty} d_{j,k}\psi_{j,k}(t)$, where $d_{j,k}=\langle f(t),\psi_{j,k}(t)\rangle$, function $f_j(t)\in W_j$, $d_{j,k}$ is the projection coefficient of $f(t)$ in W_j, j is scale shift, k is time shift. According to the wavelet theory, formulations above show that any signal $f(t)\in L^2(R)$ can be expressed by a sum of several signals with different frequency band spectra. A high- or low-frequency component of a signal $f(t)$ is in a different wavelet space. Hence the wavelet transform provide us a signal processing technique in different frequency bands. Figure 1 illustrated that a synthetic seismic signal is reconstructed from its original seismic signal that are contaminated strongly by random noise. We can observe that after the discrete wavelet transform, noise signal mainly distributes in decomposition coefficients of small scale, corresponding to high frequency components. Seismic wavelet signals almost distributed in decomposition coefficients of large or middle scales, corresponding to low or middle frequency components. Therefore, we can mute decomposition coefficients of small scales if it was less than a given small value. Through the inverse discrete wavelet transform, seismic signals are reconstructed without any noise. The same operation can perform in horizontal stacking of seismic data.

Given real random signal *x(t)* and *y(t)*, the third-order correlative function is defined as $R_{xyx}(\tau_1,\tau_2)=E[x(t)y(t+\tau_1)x(t+\tau_2)]$, which is normalized by $R_{xyx}(\tau)=\frac{R_{xyx}(\tau)}{\sqrt{R_{xx}(\tau)R_{yy}(\tau)R_{xx}(\tau)}}$, where τ_1,τ_2 are time delays and $R_{xyx}(\tau)=\sum_{\tau_2}R_{xyx}(\tau,\tau_2)$ if τ_1 is replaced by τ. Figure 2 shows the contrast between the third-order correlative function and the conventional correlative function in removing effect of correlative noise for estimation of time delay. We notice from the figure that results of the third-order correlative function present superiority over the conventional function in denosing, especially for correlative noise.

In seismic data, random noise in records of adjacent traces exhibits much coherence for which the conventional function loses its validity. Instead, the third-order correlative function can inhibit these correlative noises.

High-order correlative stacking in the wavelet domain

Supposed $x_{i,j}$ is a seismic profile in the time domain, where *i* (= 1, 2, …, M) is the trace number, *j* (= 1, 2, …, N) is a data point, $X_{i,j}^k$ is a profile in the wavelet domain, where k is a scale and the width of a window is $2P+1$. So the third-order correlative coefficient with a zero-time delay satisfies $w_{i,j}^k=\frac{1}{2P+1}\sum_{m=j-P}^{j+P}X_{i,m}^kX_{i+1,m}^kX_{i,m}^k$. $w_{i,j}^k$ is normalized by $w_{i,j}^k=\frac{w_{i,j}^k}{\sqrt{w_{i,i}^k\,w_{j,j}^k\,w_{i,i}^k}}$ (for simplicity, we used the same notation $w_{i,j}^k$ before and after the normalization). Hence a weighted stacked profile is $\overline{X}_{i,j}^k=X_{i,j}^kw_{i,j}^k$. A horizontally stacked profile with a single scale $\overline{X}_j^k=\frac{1}{M}\sum_{i=1}^{M}\overline{X}_{i,j}^k$. Then we can mute small values of $\overline{X}_j^k$ in a small scale. By the inverse wavelet transform, a final seismic profile can be obtained.

Numerical examples

Figure 3a is the common shot profile after the normal moveout correction (4 shots among 64 shots) contaminated by strong noise. Figure 3b is the conventional stacked profile. Figure 3c is the profile only stacked in wavelet domain Figure 3d is the profile compressed and stacked in wavelet domain. By comparing these figures, it is obvious that the HOCWS effectively suppressed random noises. The stacking with compression possess higher S/N ratio than that without compression.

Figure 4a is the common shot profile after the normal moveout correction (4 shots in 64 shots) with the same strong noises as figure 3a, except for extra correlative random noises are added into data below the third interface. Figure 4b is the conventional stacked profile. There is a false event produced by correlative noises. Figure 4c is the profile compressed and stacked in the wavelet domain. The false event completely suppressed in this figure.

Conclusions

Numerical examples confirm that the HOCWS is more effective than the conventional stacking for denoising, especially for suppressing correlative random noises. It can enhance S/N ratio significantly, but more computer time is needed to compute high-order correlative coefficients.

References

Robinson, J. C., 1970, Statistically optimal stacking of seismic data: Geophysics, 35,435-446.

Mao, J., 2002, optimum weighted seismic record with irregular noise: Geophysical Prospecting for Petroleum, 41,309-311.

Wang, Y., 2003, Signal noise ration weighted stacking by divided frequency band: Geophysical & Geochemical Exploration, 27,139-141

Yung, S.K., and Ikellez, L.T., 1997, An example of seismic time picking by third-order bicoherence; Geophysics, 62, 1947-1951.

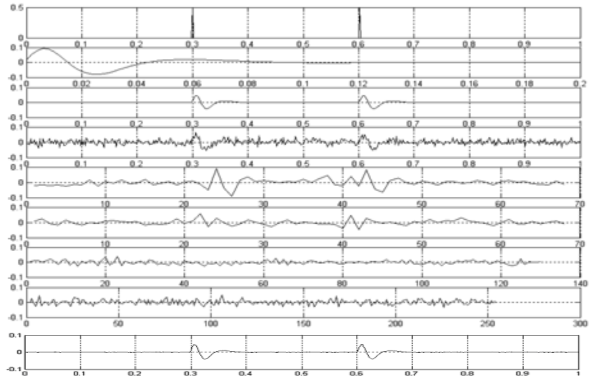
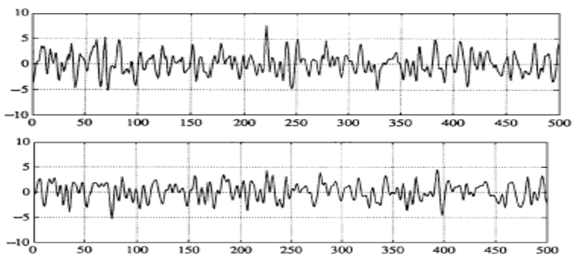
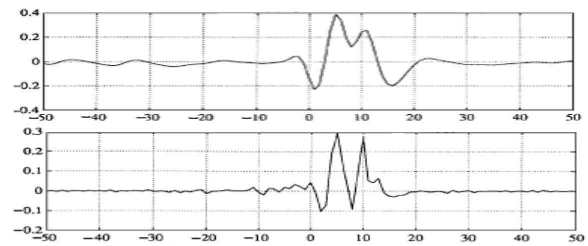


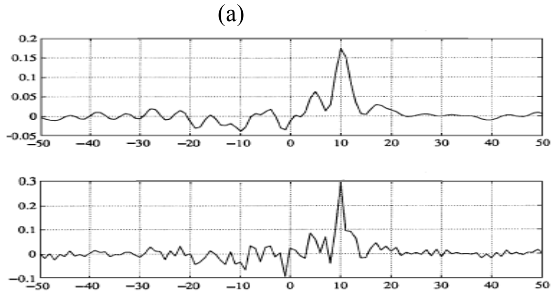
Figure 1. From the top to bottom panels: reflection coefficient, seismic wavelet, synthetic record without noise, synthetic record with noise (S/N ratio=0.5), decomposition coefficients of DWT on 3 scales corresponding to decomposition signal with different frequency and reconstructed signal after compression. The wavelet base is Daub4.



(a)

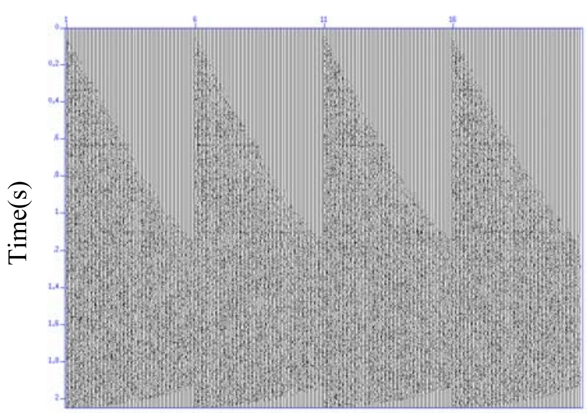


(b)

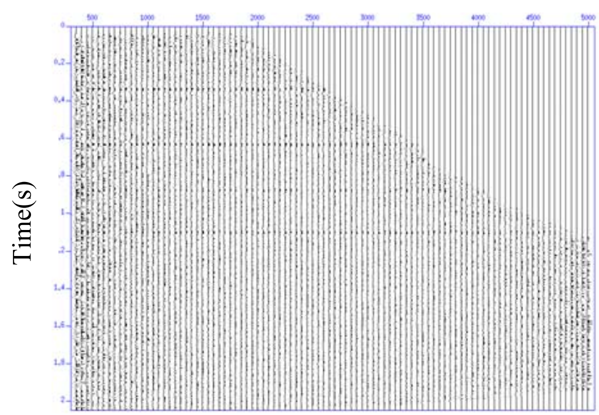


(c)

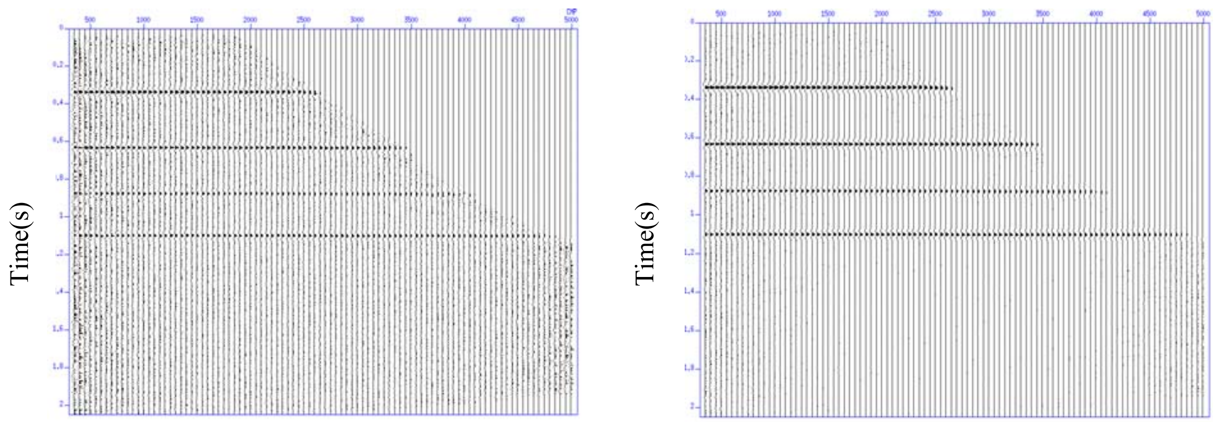
Figure 2. (a) A synthetic seismic record: the time delay of wavelet is 10 sample points and that of correlative noise is 5 sample points. (S/N ratio = 0.5). (b) The conventional correlative function and its normalized function. The last crest is caused by noises. (c) The third-order correlative function and its normalized function. The crest is only caused by noises.



(a)

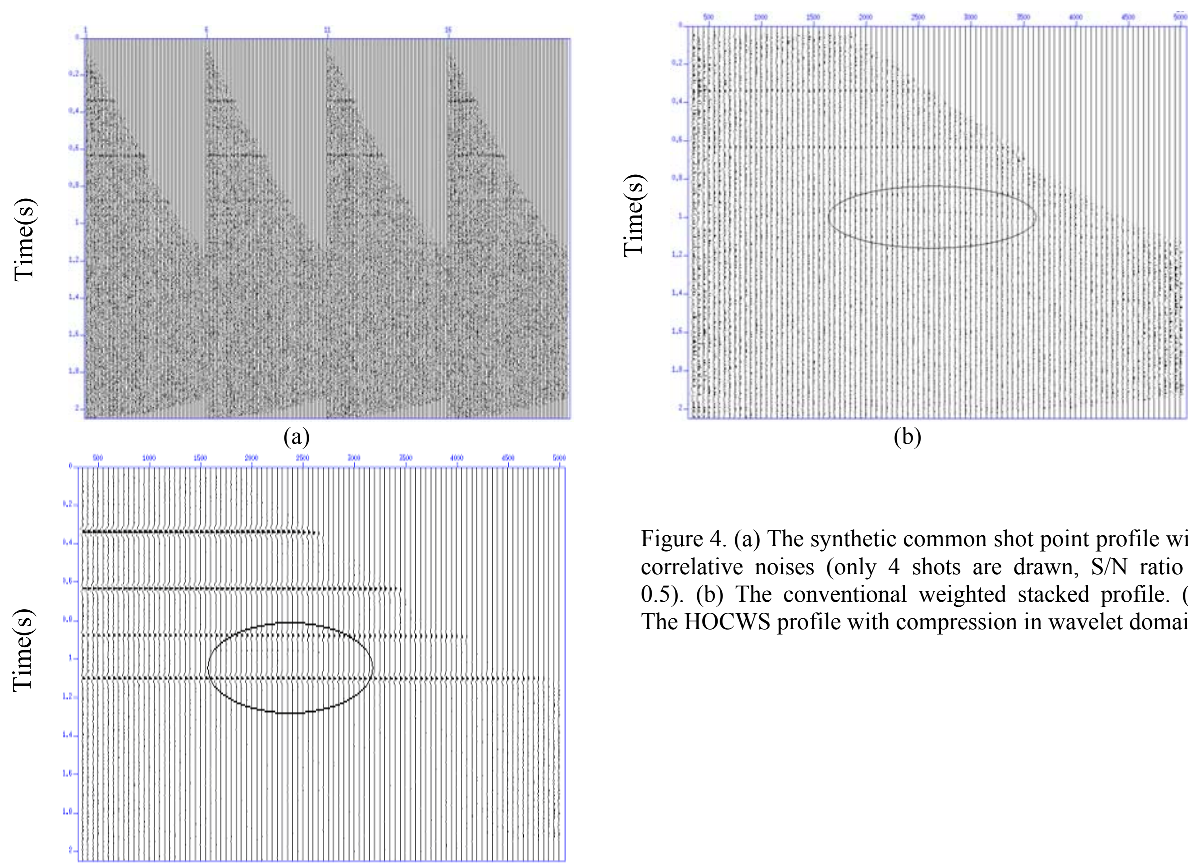


(b)



(c) (d)

Figure 3. (a) A synthetic common shot point profile with noises (only 4 shots are drawn, S/N ratio = .5). (b) The conventional weighted stacked profile. (c) The HOCWS profile without compression in wavelet domain. (d) The HOCWS profile with compression in the wavelet domain.



(a) (b)

(c)

Figure 4. (a) The synthetic common shot point profile with correlative noises (only 4 shots are drawn, S/N ratio = 0.5). (b) The conventional weighted stacked profile. (c) The HOCWS profile with compression in wavelet domain.