Utilization of high-frequency Rayleigh waves in near-surface geophysics

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Summary

The shear (S)-wave velocity of near-surface materials (such as soil, rocks, and pavement) and its effect on seismic wave propagation are of fundamental interest in many groundwater, engineering, and environmental studies. The Kansas Geological Survey conducted a three-phase research project to estimate near-surface S-wave velocity from ground roll: acquisition of ground roll, creation of algorithms that extract Rayleigh wave dispersion curves from ground roll, and development of algorithms to obtain near-surface S-wave velocity profiles. The research project has been expanded to analyze 1) the effects of higher modes on inversion of surface waves and investigation depth and 2) the feasibility of estimation of near-surface quality factor (*Q*) by inverting attenuation coefficients of Rayleigh waves. This paper summarizes these research results.

Introduction

Surface waves are guided and dispersive. Rayleigh waves are surface waves that travel along a “free” surface, such as the earth-air or the earth-water interface. Rayleigh waves are the result of interfering P and S_v waves. Particle motion of the fundamental mode of Rayleigh waves in a homogeneous medium moving from left to right is elliptical in a counter-clockwise (retrograde) direction. The motion is constrained to the vertical plane that is consistent with the direction of wave propagation. Longer wavelengths penetrate deeper than shorter wavelengths for a given mode, generally exhibit greater phase velocities, and are more sensitive to the elastic properties of the deeper layers (Babuska and Cara, 1991, 30). Shorter wavelengths are sensitive to the physical properties of surface layers. For this reason, a particular mode of surface wave will possess a unique phase velocity for each unique wavelength, hence, leading to the dispersion of the seismic signal. Ground roll is a particular type of Rayleigh wave that travels along or near the ground surface and is usually characterized by relatively low velocity, low frequency, and high amplitude (Sheriff, 1991, 143).

S-wave velocity can be derived from inverting the dispersive phase velocity of the surface (Rayleigh and/or Love) wave (Dorman and Ewing, 1962; Aki and Richards, 1980). For the case of a solid homogenous half-space, the Rayleigh wave is not dispersive and travels with a velocity of approximately 0.9194v, if Poisson’s ratio is equal to 0.25, where v is the S-wave velocity in the half space (Sheriff and Geldart, 1982, 49). However, in the case of one layer on the top of a solid homogenous half-space, dispersion of the Rayleigh wave occurs when the wavelengths of the Rayleigh wave are in the range of 1 to 30 times the layer thickness (Stokoe et al., 1994).

The Kansas Geological Survey conducted a three-phase research project to estimate near-surface S-wave velocity from ground roll (multichannel analysis of surface wave—MASW): 1) acquisition of high frequency (≥ 2 Hz) broad band ground roll, 2) creation of efficient and accurate algorithms organized in a basic data processing sequence designed to extract Rayleigh wave dispersion curves from ground roll, and 3) development of stable and efficient inversion algorithms to obtain near-surface S-wave velocity profiles. The results of this three-phase project were presented in Park et al. (1999) and Xia et al. (1999). The research project has been expanded to analyze the effects of high modes on inversion of surface waves and investigation depth (Xia et al., 2000). The research project also studied the feasibility of estimation of near-surface quality factor (*Q*) by inverting attenuation coefficients of Rayleigh waves (Xia et al., 2001). This paper will summarize these research results. Applications of MASW method in near-surface geology can be found in Miller et al. (1999).

Estimation of near-surface S-wave velocities

Park et al. (1999) and Xia et al. (1999) presented a method to estimate S-wave velocity from multichannel vertical component data. The method consists of three parts.

Surface-wave data acquisition Optimal recording of ground roll requires field configurations and acquisition parameters favorable to recording planar Rayleigh waves (Park et al., 1999). Depending on an investigation depth, Rayleigh waves with certain lengths need certain amount of time to be developed into planar waves. Plane-wave propagation of surface waves does not occur in most case until the near-offset (x_1: distance between the source and the first receiver) is great than half the maximum desired wavelength (λ_{max}) (Stokoe et al., 1994). The maximum penetration depth in a homogeneous medium is about one wavelength. The current accepted rule of thumb of the maximum penetration depth is approximately half the longest wavelength (Rix and Leipski, 1991). The near-offset distance should be selected as almost the same as investigation depth.

High-frequency surface waves attenuate quite rapidly with distance away from source. To acquire dominated high-frequency components in the further offsets, the far-offset (the distance between the source and the furthest receiver) is normally selected as a distance that is twice the investigation depth. To avoid spatial aliasing, receiver spacing should not be larger than half the shortest wavelength measured.

Dispersion curves Dispersion curves can be obtained by the following transformation (Park et al., 1998). Considering offset-time (*x-t*) domain representation $u(x,t)$ of a shot gather, the Fourier transformation can be applied to time axis of $u(x,t)$ to obtain $U(x,w)$ and applying the following integral transformation to $U(x,w)$ we obtain $V(\phi,w)$:

$$V(\phi,w)=\int e^{i\phi x}\left[U(x,w)\big/\left|U(x,w)\right|\right]dx=\int e^{-i(\Phi-\phi)x}\left[A(x,w)\big/\left|A(x,w)\right|\right]dx \qquad (1)$$

where $\Phi = w/c_w$, *w* is frequency in radian, c_w is phase velocity for frequency *w*, and *A(x, w)* is amplitude spectrum. Dispersion curves are obtained by transforming $V(w,\phi)$ to $I(w,c_w)$ through changing the variables such that $c_w = w/\phi$. In the $I(w,c_w)$ wavefields, there will be peaks along the c_w-axis. The locus along these peaks over different values of *w* permits the images of dispersion curves to be constructed.

Inversion of dispersion curves Rayleigh wave phase velocity of a layered earth model is a function of frequency and four groups of earth properties: P-wave velocity, S-wave velocity, density, and thickness of layers. Analysis of the Jacobian matrix provides a measure of dispersion curve sensitivity to earth properties (Xia et al., 1999). S-wave velocity is the dominant influence on a dispersion curve in a high frequency range (≥ 2 Hz) so only S-wave velocities are unknowns in our inversion. An iterative solution technique to the weighted equation proved very effective in the high frequency range when using the Levenberg-Marquardt (L-M) method (Marquardt, 1963). Convergence of the weighted solution is guaranteed through selection of the damping factor using the L-M method.

S-wave velocities (earth model parameters) can be represented as the elements of a vector $\mathbf{x}$ of length *n*, $\mathbf{x} = [v_{s1}, v_{s2}, v_{s3}, ..., v_{sn}]^T$. Similarly, the measurements (data) of Rayleigh wave phase velocities at *m* different frequencies can be represented as the elements of a vector $\mathbf{b}$ of length *m*, $\mathbf{b} = [b_1, b_2, b_3, ..., b_m]^T$. After linearizing a model response by Taylor-series expansion, we obtain:

$$\boldsymbol{J}\ \Delta\boldsymbol{x} = \Delta\boldsymbol{b}, \qquad (2)$$

where $\Delta\boldsymbol{b}$ [$= \mathbf{b} - \mathbf{c}_R(\mathbf{x}_o)$] is the difference between measured data and model response to the initial estimation, $\mathbf{c}_R(\mathbf{x}_o)$ is the model response to the initial S-wave velocity estimates, $\mathbf{x}_o$; $\Delta\boldsymbol{x}$ is a modification of the initial estimation; $\boldsymbol{J}$ is the Jacobian matrix with *m* rows and *n* columns ($m > n$). The elements of the Jacobian matrix are the first order partial derivatives of $\mathbf{c}_R$ with respect to S-wave velocities. Since the number of data points contained in the dispersion curve is generally much larger than the number of layers used to define the subsurface ($m > n$), equation (2) is usually solved by optimization techniques. The objective function is

$$\Phi = \left\|\boldsymbol{J}\Delta\boldsymbol{x}-\Delta\boldsymbol{b}\right\|_2 \boldsymbol{W} \left\|\boldsymbol{J}\Delta\boldsymbol{x}-\Delta\boldsymbol{b}\right\|_2 + \alpha\left\|\Delta\boldsymbol{x}\right\|_2^2, \qquad (3)$$

where $\|\ \|_2$ is the l_2-norm length of a vector, α is the damping factor, and $\boldsymbol{W}$ is a weighting matrix. This is a constrained (weighted) least-squares problem. The weighting matrix for our inversion is based on differences in Rayleigh wave phase velocities with respect to frequency. Since the weighting matrix $\boldsymbol{W}$ (Equation 3) is both diagonal and positive, we can write $\boldsymbol{W} = \boldsymbol{L}^T \boldsymbol{L}$, where $\boldsymbol{L}$ is also a diagonal matrix.

Employing the singular value decomposition (SVD) technique (Golub and Reinsch, 1970) to minimize the objective function (3) allows us to change the damping factor (α) without recalculating the inverse matrix of ($\boldsymbol{A}^T \boldsymbol{A} + \alpha \boldsymbol{I}$), where $\boldsymbol{A} = \boldsymbol{L}\,\boldsymbol{J}$. The solution is

$$\Delta\boldsymbol{x} = \boldsymbol{V}\left(\Lambda^2 + \alpha\boldsymbol{I}\right)^{-1}\Lambda\boldsymbol{U}^T\boldsymbol{d} \qquad (4)$$

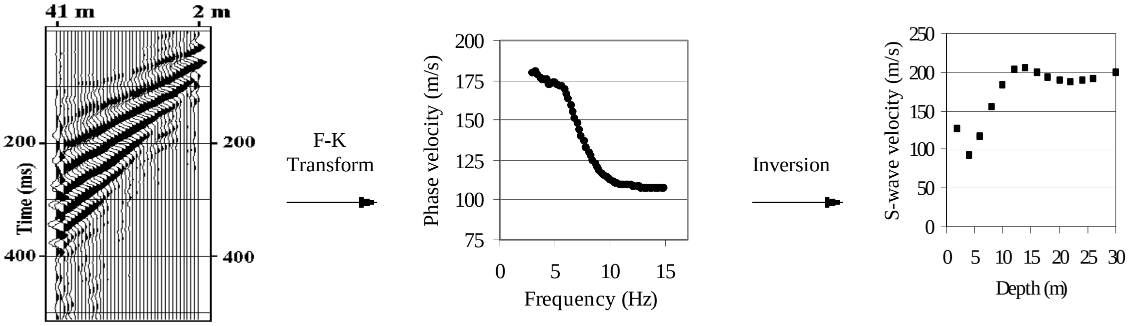
where matrix $\boldsymbol{A}$ is decomposed as $\boldsymbol{A} = \boldsymbol{U}\Lambda\boldsymbol{V}^T$, $\boldsymbol{d} = \boldsymbol{L}\boldsymbol{b}$, and $\boldsymbol{I}$ is the unit matrix.

Figure 1 shows the processing flow from a shot gather to an S-wave velocity profile. S-wave velocity profiles (S-wave velocity vs. depth) derived from MASW compared favorably to direct borehole measurements at sites in Kansas, Wyoming, and British Columbia (Xia et al., 2002). On the average, the difference between MASW calculated Vs and borehole measured Vs is less than 15%.

Advantages of using high modes

A series of Rayleigh waves of different frequencies can have the same wave velocity. These different frequency Rayleigh waves for a given phase velocity are known as modes and are characterized by their different number of horizontal nodal planes (planes of no particle displacement within the layer) (Garland, 1979, 60). In other words, more than one phase velocity can be associated with a given frequency of Rayleigh wave simply because these waves can travel at different velocities for a given frequency. The lowest velocity for any given frequency is called the fundamental mode velocity (or the first mode). The next velocity higher than the fundamental mode phase velocity is called the second mode velocity, and so on. All phase velocities that are higher than the fundamental mode velocities are called higher modes.

Xia et al. (2000) have shown that the inversion process is more stable when higher mode data are included in the inversion process. Experimental analysis indicates that energy of higher modes tends to become more dominant as the source distance becomes greater. In some cases, higher mode data are necessary since shorter wavelength components of fundamental mode Rayleigh waves are obscured by these higher frequency data where higher modes of Rayleigh waves dominate. By analyzing the Jacobian matrix of the inversion system with high modes, Xia et al. (2000) concluded that higher mode data have a deeper investigation depth than fundamental mode data do. Their modeling results and real world examples showed that higher mode data stabilize the inversion procedure and increase the resolution of inverted S-wave velocities. Their research is the first attempt to utilize properties of higher modes to obtain near-surface S-wave velocities by inverting high-frequency surface wave data.



Multichannel raw field data

Dispersion curve of Rayleigh wave

Shear-wave velocity profile

Figure 1. A diagram of the MASW method. Multi-channel raw field data, which contain enhanced ground roll signals, are acquired. Rayleigh wave phase velocity is extracted from the field data in the F-K domain. The phase velocity, finally, is inverted for a shear-wave velocity profile (V_s vs depth).

Near-surface *Q*

The quality factor as a function of depth is of fundamental interest in groundwater, engineering, and environmental studies, as well as in oil exploration and earthquake seismology. A desire to understand the attenuative properties of the earth is based on the observations that seismic wave amplitudes are reduced as waves propagate through an elastic medium. For a plane wave traveling in a homogeneous medium, the quality factor Q is determined by (Johnston and Toksöz, 1981)

$$Q = \frac{\pi f}{\alpha v} \tag{5}$$

where v, f, and α are the velocity, the frequency, and the attenuation coefficient of the plane wave, respectively. To determine Q as a function of depth in near-surface materials, the assumption of homogeneity is no longer valid because of complexity of the near-surface geology. Utilization of high-frequency Rayleigh waves is essential in finding the quality factors of near-surface materials. The relationship between Rayleigh wave attenuation coefficients and the quality factors for P and S waves of a layered model were given by Anderson et al. (1965) as:

$$\alpha_R(f) = \frac{\pi f}{C_R^2(f)} \left[\sum_{i=1}^{n} P_i(f) Q_{Pi}^{-1} + \sum_{i=1}^{n} S_i(f) Q_{Si}^{-1} \right]. \tag{6}$$

where

$P_i(f) = V_{Pi} \dfrac{\partial C_R(f)}{\partial V_{Pi}}$, $S_i(f) = V_{Si} \dfrac{\partial C_R(f)}{\partial V_{Si}}$, and $\alpha_R(f)$ is Rayleigh wave attenuation coefficients in 1/length, and f is frequency in Hz. Q_{Pi} and Q_{Si} are the quality factors for P and S waves of the ith layer, respectively; V_{Pi} and V_{Si} are the P-wave velocity and S-wave velocity of the ith layer, respectively; $C_R(f)$ is Rayleigh wave phase velocity; and n is the number of layers of a layered earth model.

The modeling results (Xia et al., 2001) suggest that it is feasible to solve for P-wave quality factor Q_P and S-wave quality factor Q_S in a layered earth model by inverting Rayleigh wave attenuation coefficients when Vs/Vp reaches 0.45. Only Q_S can be estimated from Rayleigh wave attenuation coefficients when Vs/Vp is less than 0.45. We used an algorithm from Menke (1984) including introduction of a damping factor to solve quality factors Q_{Pi} and/or Q_{Si} based on Equation (6). The sensitivity analysis showed that errors in inverted quality factors can reach 1 to 1.5 times the error in attenuation coefficients. Compared to the inversion system (Equation 3) that Xia et al. (1999) developed to invert S-wave velocities from Rayleigh wave phase velocities (10% error in surface wave phase velocity will result in 6% error in S-wave velocity), the inversion system based on Equation (6) possesses less stability. Hence, accurate calculation of Rayleigh wave attenuation coefficients is critical. On the other hand, the inversion system based on Equation (6) is more stable than AVO (amplitude versus offset) analysis studied and practiced in the oil industry for the last 20 years (Hilterman, 2001). Jin et al. (2000) concluded that in AVO analysis a 10% error in incident angles could result in a 40% error in reflection coefficients.

Conclusions

Inversion of Rayleigh waves can provide reliable S-wave velocity profile (± 15%). The inversion system is numerically stable. An inversion system with high mode data can provide even high stability and high resolution in the inverted model. After estimation of S-wave velocity, it is feasible to determine near-surface quality factors Q_p and/or Q_S.

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