GEOPHYSICS, VOL. 56, NO. 4 (APRIL 1991); P. 537-541, 8 FIGS., 3 TABLES.



# Correction of topographic distortion in gravity data

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## ABSTRACT

The numerical equivalent source is used to correct distortions in gravity anomalies caused by topographic relief. The error involved in the topographic correction is a function of the depth of the equivalent source. The optimum depth of the equivalent source is that which maximizes the smoothness of the calculated anomaly between control points. The technique is straightforward and can be incorporated easily into a standard data processing sequence.

## INTRODUCTION

It is well known that topographic relief at the surface of measurement distorts the gravity anomaly. This distortion is due to the varying vertical separation of the measurement points from the source body and is not accounted for by the standard Bouguer and free-air corrections. Tsuboi (1965) calls such anomalies "station" Bouguer anomalies reduced to sea level and distinguishes them from real Bouguer anomalies at sea level. The latter involves a vertical (upward and/or downward) continuation of the gravity field onto a common horizontal plane, which we call topographic correction. Figure 1 is an example. Figure 1B shows a cross-section, with a station spacing of 100 m, through a point-mass gravity source buried 100 m directly beneath a 100-m vertical scarp at the surface. The source has an excess mass of $10^{10}$ kg. Figure 1A plots the true anomaly if measurements were taken on a horizontal datum 200 m above the source ($z = -100$ m) as well as the "measured" anomaly at the topographic surface, after "normal" data corrections to a common datum have been performed. The distortion in the measured anomaly is due to the decreased vertical separation between the source body and the lower measuring stations.

Several methods of correcting the topographic distortion have been developed. Dampney (1969) determined an equivalent source of discrete point masses on a horizontal plane from Bouguer anomaly measurements on an irregular surface by solving a system of simultaneous equations. He found that the depth to the equivalent source should be limited within a certain range relative to the station spacing by studying the condition number of the matrix of the system. Henderson and Cordell (1971) discussed an approach of topographic correction by means of finite harmonic series. Syberg (1972) developed simple convolution operators for upward continuation of potential field data from a general surface to a horizontal plane. Bhattacharyya and Chan (1977) determined an equivalent source by solving a Fredholm integral equation of the second kind.

We present an alternative correction which is effective and efficient enough to be incorporated into a routine processing stream. The corrected anomaly is calculated from an ensemble of point-mass equivalent sources derived from the iterative solution of the Dirichlet boundary-value problem. The optimum depth to the source ensemble is that which maximizes the smoothness of the calculated anomaly between the data points.

## THE METHOD

We seek the gravity function $U$ in the region $z \leq f(x, y)$ ($z$ is positive downward), such that

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0 , \qquad (1)$$

$$U|_{z=f(x,y)} = g(x,y),$$

where $f(x, y)$ is the topographic function and $g(x, y)$ is the measured gravity anomaly. With discrete functions $f(x, y)$ and $g(x, y)$, we have a Dirichlet boundary-value problem.

An iterative forward solution for $U$ can be obtained using an equivalent source approximation. The equivalent source is a collection of point masses, one beneath each surface measurement point. There are two primary difficulties with the equivalent source approach, both related to the depth of the source ensemble. If the source depth is too shallow relative to the station spacing, then the anomaly at each station is determined only by the source directly beneath it. The solution converges quickly, but the anomaly tends to disappear or "sag" between the stations, i.e., high-frequency noise (HFN) is intro-

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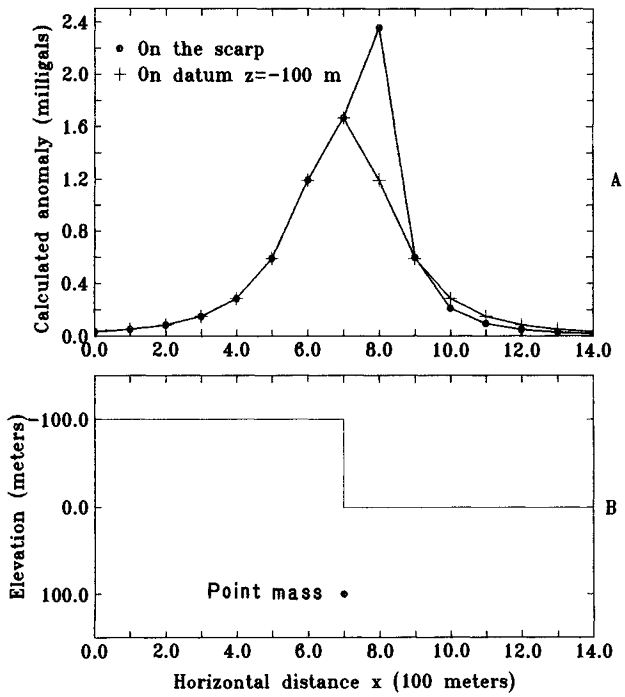


FIG. 1. A vertical cross-section through a point-mass gravity source buried 100 m directly beneath a 100-m vertical scarp at the surface (B) and the Bouguer anomalies on the scarp and on the horizontal datum 200 m above the source ($z = -100$ m) (A).

duced. This noise appears when the equivalent source is used to calculate the anomaly on a new surface (the purpose of the operator.) High-frequency noise is also introduced if the source ensemble is too deep because positive and negative sources must be combined to correct for anomaly width. With sources that are too deep, the solution converges slowly or not at all. The present work demonstrates that an optimum source depth can be determined which minimizes HFN between stations and therefore minimizes noise introduced by the topographic operator.

Figure 2 illustrates the HFN problem in two dimensions (2-D). The "measured" anomaly is specified at 11 stations, located at the horizontal distances given by $x = 0.0$ to 10.0 (station spacing DX = 1). The equivalent source ensemble is 11 mass lines (because the problem is 2-D), one beneath each station. The unknowns are the mass per unit length of each mass line, which is determined by 11 simultaneous equations. Based on their condition numbers, the equations are well posed for the depths to the equivalent source in the region from 0.1 DX to 10 DX. Therefore, the unknowns can be solved exactly, i.e., the difference between the "measured" anomaly and the anomaly caused by the equivalent source can be made arbitrarily small for depths of the equivalent source in the region 0.1 DX to 10 DX. HFN due to inappropriate source depth is demonstrated by using the derived equivalent source to calculate the anomaly at halfway positions ($X = 0.5$, 1.5, etc.). Figure 2 shows the disappearance of the anomaly at intermediate points when the source depth is very shallow ($H = 0.1$ DX) and also shows distortion of the anomaly when the source depth is too deep ($H = 10.0$ DX). Figure 2 also demonstrates the existence of an optimum source depth that minimizes halfway point HFN — is contrary to intuition, which suggests the optimum source depth is the maximum source depth that can be brought to convergence.

To determine the optimum depth for the equivalent source, we quantitatively estimate the degree of smoothness with equation (2):

$$S(H) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\bar{g}_{i,i+1} - g_{i+0.5})^2}\,, \tag{2}$$

where $S$ is our estimate of smoothness, $H$ is the depth to the equivalent source, $n$ is the number of halfway calculation points, $\bar{g}_{i,i+1}$ is the average of the anomaly values calculated at stations $i$ and $i+1$, and $g_{i+0.5}$ is the calculated gravity from the equivalent source at the point halfway between $i$ and $i+1$. This is a valid estimator as long as the average curvature of the anomaly between points i and i+1 is roughly zero. A more rigorous treatment could use a higher order of polynomial interpolation function. $S(H)$ is calculated for the equivalent source ensemble as $H$ is increased from some small value (e.g., 0.5 DX). The optimum depth $H_j$ is that which satisfies the inequality $S(H_{j-1}) > S(H_j) < S(H_{j+1})$. Figure 2 suggests that the optimum depth is in the neighborhood of $H$ = DX.

The appropriate mass distribution for the equivalent source at a given depth is determined by iteratively reducing the misfit between the measured anomaly and the calculated anomaly at the measurement stations. The initial estimate of the mass distribution is given by the Gauss formula (Garland, 1979, p. 136),

$$M_i^0 = g_i \, \mathrm{DS}/2\pi G \tag{3}$$

where $M_i^0$ is the initial value of the mass of the $i$th source element, $g_i$ is the anomaly at the $i$th station, $G$ is the gravitational constant, and DS is the average area between data points. Forward calculations are performed to determine the $E_{\mathrm{rms}}$ error in the equivalent gravity field relative to $g(x, y)$ by

$$E_{\mathrm{rms}} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (g_i^k - g_i)^2}\,, \tag{4}$$

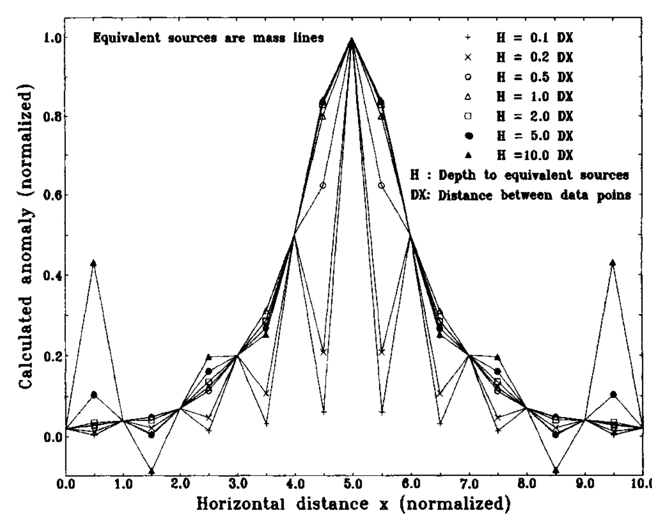


FIG. 2. High-frequency noise of intermediate-point anomalies as a function of depth of the equivalent source. Intermediate-point anomalies are inaccurate for source depths that are both too shallow and too deep.

where $m$ is the total number of data points, $g_i^k$ is the value of the equivalent gravity field at the $i$th point after the $k$th iteration, and $g_i$ is the measured gravity at the $i$th point. Iterative reduction in the $E_{rms}$ error is then obtained by adjusting the mass distribution. The $i$th point mass at the $(k+1)$th iteration is given by

$$M_i^{k+1} = M_i^k + C(g_i - g_i^k)\,H^2/G, \tag{5}$$

where $H$ is the depth to equivalent source and $C$ is a coefficient chosen in the range from 0.1 to 1 to produce convergence if $H$ is near to the optimum depth. Formula (5) is derived from the differential formula of the gravity field of a point mass. Initially, $C$ is set to 1 which gives the maximum modification to the equivalent source. If this value of $C$ does not reduce $E_{rms}$ error, then $C$ is repeatedly halved until iterative reduction in $E_{rms}$ error is achieved. Iteration with the appropriate $C$ value continues until the $E_{rms}$ error in the equivalent field is less than or equal to the precision of the gravity data. The gravity anomaly of a point source satisfies Laplace's equation; thus, the solution obtained above satisfies all conditions of the Dirichlet boundary-value problem.

## DEMONSTRATION USING SYNTHETICS

Table 1 lists the results of topographic corrections to the gravity fields of eight buried rectangular slabs with a unit density and having the same horizontal boundaries $x_1 = 1000$ m, $x_2 = 1800$ m, $y_1 = 600$ m, $y_2 = 1200$ m. The depth to top and bottom ($z_1$ and $z_2$, respectively) of the slabs is varied as given in Table 1. The "measured" anomaly is calculated at $z = 0$ m, while the true anomaly is calculated at $z = -50$ m. The equivalent source is then used to continue the measured anomaly to $z = -50$ m where the misfit with the exact solution is evaluated. The station grid is 15 × 15 with a station spacing of 200 m in both $x$ and $y$ directions. The error between the true anomaly and the corrected anomaly is the true error and is calculated with

$$E_T = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(ge_i - gc_i)^2}, \tag{6}$$

where $ge_i$ and $gc_i$ are the true anomaly and the anomaly calculated from the equivalent source on the datum $z = z_0$, respectively. The solution is iterated until $E_{rms}$ is less than 0.05 mGal. A perfect estimator for $S$ would show minima of $S$ and $E_T$ at the same depth of equivalent source. Table 1 indicates that this is only approximately true for the estimator used here. Even so, the value of $E_T$ (Table 1, column 5) at minimum $S$ ($S_{min}$) is very close to $E_{rms}$, indicating that very little noise is introduced into the continuation when the equivalent source at $H(S_{min})$ is used. This approach to the topographic correction is thus stable and reliable. For upward continuation, $E_T$ (Table 1, column 5) continues to decrease with increase in equivalent source depth beyond the depth of $S_{min}$. This is expected because upward continuation reduces the HFN introduced by using a source that is too deep (Figure 2). Downward continuation of the equivalent source results demonstrates (Table 1, column 7) the expected correspondence between the depth of $S_{min}$ and the depth of minimum $E_T$ (Table 1, column 7).

The technique is applied to the step topography problem of Figure 1. The grid is 15 × 15 with a station spacing of 100 m in both $x$ and $y$ directions. Table 2 lists the result. Figure 3 plots the anomaly after topographic correction, which is calculated from the ensemble of equivalent sources located at the optimum depth ($H$ = 100 m). Figure 4 shows the anomaly which is calculated from the ensemble of equivalent sources located at the depth 12.5 m. Clearly, the distortions in the anomaly still remain in the latter case even though $E_{rms}$ error is equal to 0.02 mGal.

**Table 1. The results of testing the technique.**

| $z_1/z_2$ (m) | $H$ (m) | $k$ | $E_{rms}$ | $E_T$ ($z=-50$ m) | $S$ | $E_T$ ($z=10$ m) |
|---|---|---|---|---|---|---|
| 50/2000 | 100 | 17 | 0.0413 | 0.5704 | 0.8654 | 0.2796 |
| | 200 | 8 | 0.0483 | 0.0644 | 0.0564 | 0.0469 |
| | 400 | 59 | 0.0495 | 0.0349 | 0.0743 | 0.0531 |
| 100/2000 | 100 | 17 | 0.0389 | 0.5226 | 0.7874 | 0.2511 |
| | 200 | 7 | 0.0481 | 0.0608 | 0.0510 | 0.0475 |
| | 400 | 36 | 0.0488 | 0.0346 | 0.0605 | 0.0526 |
| 200/2000 | 100 | 16 | 0.0466 | 0.3921 | 0.6100 | 0.2829 |
| | 200 | 6 | 0.0420 | 0.0547 | 0.0421 | 0.0415 |
| | 400 | 21 | 0.0490 | 0.0380 | 0.0472 | 0.0520 |
| 400/2000 | 100 | 15 | 0.0453 | 0.3510 | 0.5016 | 0.1329 |
| | 200 | 5 | 0.0338 | 0.0473 | 0.0320 | 0.0333 |
| | 400 | 12 | 0.0476 | 0.0399 | 0.0394 | 0.0497 |
| | 800 | 22 | 0.0485 | 0.0402 | 0.0513 | 0.0505 |
| 600/2000 | 100 | 14 | 0.0456 | 0.2114 | 0.3198 | 0.1822 |
| | 200 | 4 | 0.0390 | 0.0492 | 0.0363 | 0.0380 |
| | 400 | 9 | 0.0416 | 0.0371 | 0.0352 | 0.0429 |
| | 800 | 19 | 0.0477 | 0.0470 | 0.0450 | 0.0480 |
| 50/1000 | 100 | 15 | 0.0489 | 0.4566 | 0.7052 | 0.2183 |
| | 200 | 8 | 0.0474 | 0.0537 | 0.0528 | 0.0456 |
| | 400 | 57 | 0.0497 | 0.0338 | 0.0727 | 0.0533 |
| 10/100 | 100 | 8 | 0.0468 | 0.0999 | 0.1605 | 0.0695* |
| | 200 | 7 | 0.0435 | 0.0449 | 0.0337 | 0.0422 |
| | 400 | 44 | 0.0492 | 0.0384 | 0.0373 | 0.0502 |
| 20/500 | 100 | 14 | 0.0412 | 0.3212 | 0.5267 | 0.2338 |
| | 200 | 9 | 0.0446 | 0.0531 | 0.0520 | 0.0413 |
| | 400 | 81 | 0.0498 | 0.0366 | 0.0773 | 0.0525 |

Note $z_1$ and $z_2$ are the depths to the top and the bottom of the rectangular slabs, respectively; $H$ is the depth to the equivalent source; $k$ is the number of iterations; the definitions of $S$, $E_{rms}$, and $E_T$ are shown in the equations (2), (4), and (6), respectively; $z$ is the elevation of the corrected datum.
*The datum $z$ in this case is 5 m.

**Table 2. The results of the point source problem.**

| $H$ (m) | $k$ | $E_{rms}$ | $E_T$ ($z=-100$ m) | $S$ |
|---|---|---|---|---|
| 12.5 | 86 | 0.0200 | 0.0726 | 0.1380 |
| 50 | 9 | 0.0183 | 0.0257 | 0.0459 |
| 100 | 20 | 0.0239 | 0.0244 | 0.0299 |
| 200 | 20 | 0.0541 | 0.0426 | 0.0441 |

## McPHERSON COUNTY, KANSAS

We now topographically correct the gravity of McPherson County, Kansas, shown after removal of a second-order regional trend in Figure 5. The data are gridded to 1.609 km by 1.609 km by Surface III (Sampson, 1989), giving a total of 1156 grid points. Total relief in McPherson County is 124 m, as plotted in Figure 6. The selected horizontal datum is 500 m above sea level ($z = -500$ m). Table 3 shows the values of smoothness

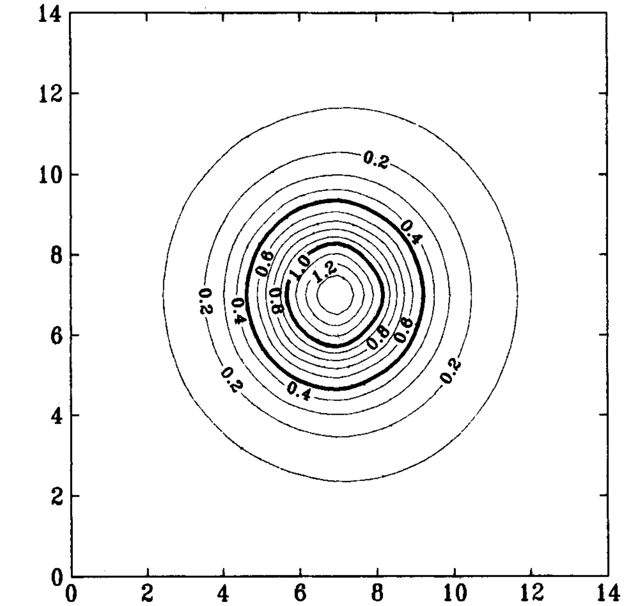


FIG. 3. Anomaly on the horizontal datum $z = -100$ m caused by an ensemble of point-mass equivalent sources located at a depth of 100 m. $E_{\text{rms}}$ (the error between the "measured" anomaly on the scarp and the anomaly caused by the equivalent source on the same surface) is 0.0239 mGal. $S$ (smoothness between the data points) is 0.0299 mGal. $E_T$ (the actual error in the corrected gravity at the datum) is 0.0244 mGal. The topographic correction is visually satisfying. The unit in both $x$ and $y$ directions is a station spacing, 100 m.

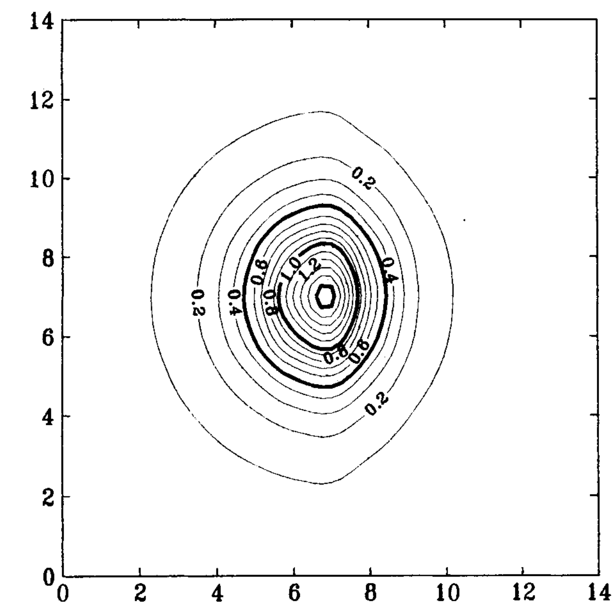


FIG. 4. Anomaly on the horizontal datum $z = -100$ m caused by the ensemble of point-mass equivalent sources located at a depth of 12.5 m. $E_{\text{rms}}$ is equal to 0.02 mGal, essentially the same as for Figure 3. $S$ is equal to 0.1380 mGal, much larger than in Figure 3. $E_T$ is 0.0726 mGal. The unit in both $x$ and $y$ directions is a station spacing, 100 m.

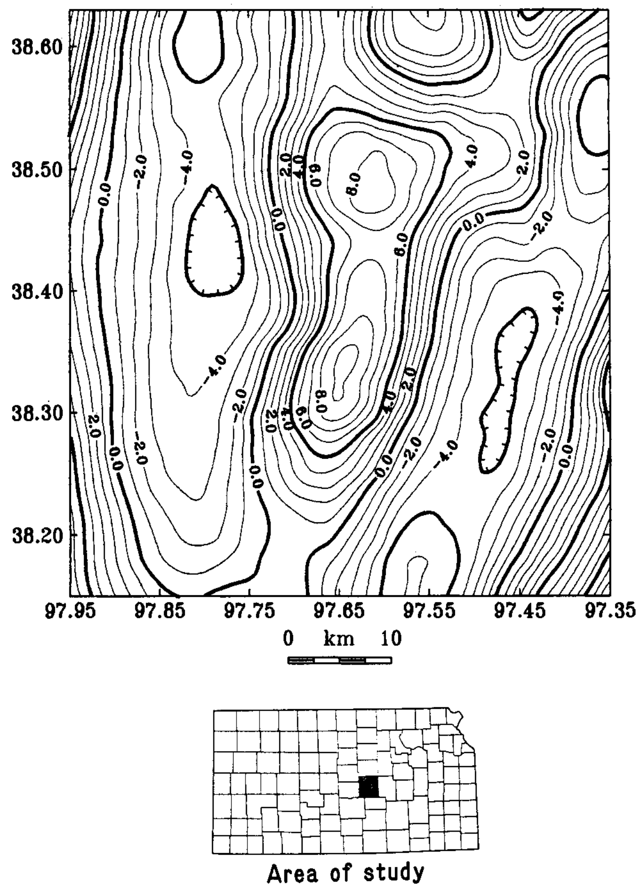


FIG. 5. Residual Bouguer anomaly of McPherson County, Kansas, after removal of a second-order trend. Contour interval is 1 mGal. Coordinates in Figures 5–8 are latitudes and longitudes.

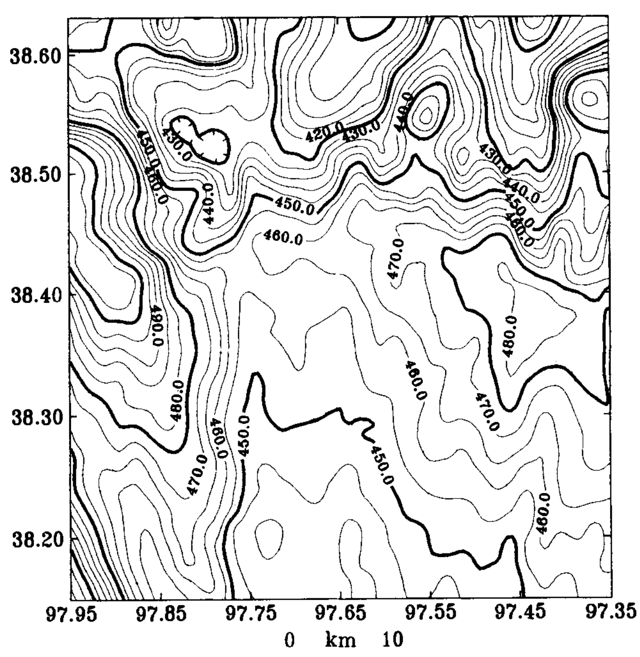


FIG. 6. The topographic map of McPherson County, Kansas. The contour interval is 5 m.

**Table 3. The results of the example of McPherson County, Kansas.**

| $H$ (m) | $k$ | $E_{rms}$ | $E_T$ ($z = -500$ m) | $S$ |
|---|---|---|---|---|
| 804.5 | 7 | 0.0598 | | 3.1573 |
| 1609.0 | 5 | 0.0684 | N/A | 0.2387 |
| 2413.5 | 10 | 0.0958 | | 0.1106 |
| 3218.0 | 15 | 0.2207 | | 0.1429 |

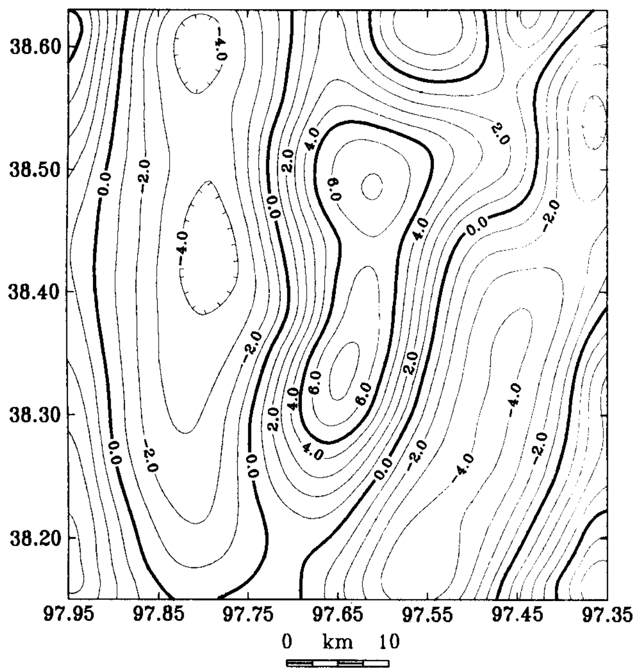


FIG. 7. Topographically corrected anomaly on the horizontal datum $z = -500$ m, calculated from the equivalent source located at a depth of 2413.5 m. $E_{rms}$ is 0.0958 mGal. $S$ is 0.1106 mGal.

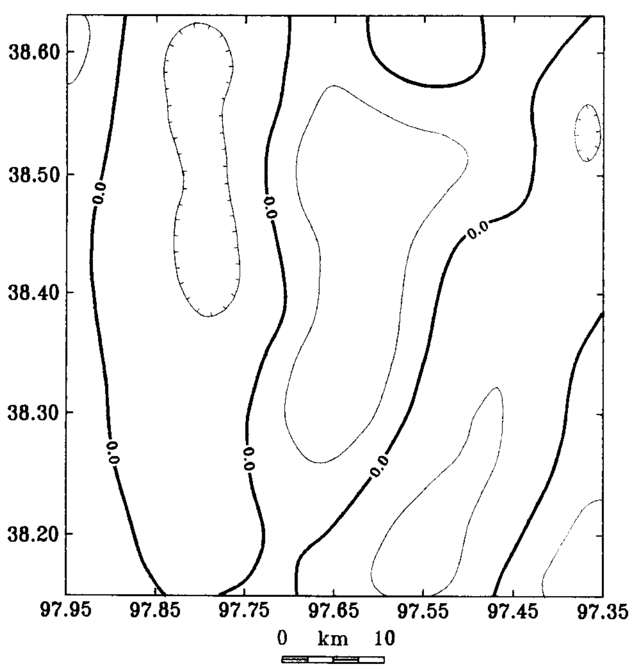


FIG. 8. Anomaly calculated from an equivalent source at a depth of 804.5 m. $E_{rms}$ is 0.0598 mGal. $S$ is 3.1573 mGals. The correction is clearly inappropriate, as suggested by the non-minimum value of $S$.

$S$ as a function of depth to equivalent source $H$, which indicate that the optimum depth to the equivalent source is 2413.5 m (1.5 DX). Figure 7 gives the anomaly after the topographic correction. $E_{rms}$ error is less than 0.1 mGal, the precision of Bouguer anomaly, after ten iterations. Although the $E_{rms}$ error is equal to 0.06 mGal in Table 3 when $H = 804.5$ m after seven iterations, Figure 8 indicates a poor topographic correction with $H = 804.5$ m. Minimization of $S$ rather than $E_{rms}$ error defines the optimum source depth.

## DISCUSSION

The numerical results in Table 1 show that the number of iterations required for convergence increases significantly as the equivalent source becomes too deep. On the other hand, few iterations are required when the equivalent source is at the optimum depth. Thus, slow convergence of the $E_{rms}$ error (single-step $\Delta E_{rms} \approx 1$ percent of the last $E_{rms}$) is an indication that the equivalent source is too deep.

The minimization of HFN input to the data by optimization of equivalent source depth permits reasonable downward continuation. Thus, the corrected datum need not be the highest point of the measurement surface.

For larger data sets, calculation of the optimum depth to equivalent source can be prohibitive. Considerable time savings can be realized by using a Taylor series to calculate approximate anomalies and by neglecting the contributions of sources that are very distant from the point of measurement. At horizontal distance ten times the depth, the gravity contribution is only 0.1 percent. Care must be taken in this regard, however, because the topographic corrections are also small relative to the total gravity.

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