A "New" Definition of Wavelet Energy and Implications on the Determination of Resolving Power

Ralph W. Knapp, Kansas Geological Survey

SUMMARY

The use of a definition of wavelet energy that considers total energy of the seismic signal rather than only the energy of the real component of the wavelet results in a proof that wavelets that have a common amplitude envelope also share resolving power. These are wavelets that differ only by a constant or linear phase shift and are termed here a wavelet class. Total instantaneous wavelet energy is proportional to the square of the amplitude envelope as determined by complex trace analysis. Thus wavelets that share an amplitude envelope (the wavelet class) have the same resolving power because total energy distribution is identical.

Zero-phase wavelets and others of its class, such as the quadrature-phase wavelet, have maximum possible resolving power within a suite of wavelets. (A suite includes those wavelets that have a common amplitude spectrum.) Extant definitions of resolving power use only the real attribute of the wavelet and, thus, favor zero-phase wavelets over others in its class. The proof that zero-phase class wavelets have maximum resolving power also implies that the minimum-phase wavelet has the greatest resolving power of any causal wavelet in a suite.

INTRODUCTION

The energy of a wavelet is defined here to be the total energy of the seismic signal, the sum of kinetic and potential energy. This is contrary to prior definitions of wavelet energy which only use the amplitudes of the real component of the wavelet (for examples, Robinson, 1964; Robinson and Treitel, 1980; Berkhout, 1974 and 1984; and Widess, 1984). Total instantaneous energy is proportional to the square of the amplitude envelope. The amplitude envelope is defined by complex trace analysis (Taner, et al., 1979).

The shortest possible wavelet energy distribution has the greatest resolving power (Berkhout, 1974 and 1984; Schoenberger, 1974; Clearbout, 1985; and Knapp, 1990). That is, the wavelet that concentrates its energy most efficiently about some time point has the greatest resolving power. Consequently, wavelets with identical wavelet envelopes have the same resolving power; i.e., the same distribution of total energy. (Kallweit and Wood (1982) discuss resolving power very nicely.)

A suite of wavelets is defined by Robinson and Treitel (1965, 1980) as being those wavelets that share a common amplitude spectrum. Within a suite it is found that wavelets of constant-angle phase shift have identical wavelet envelopes. Common examples are the zero-phase wavelet and the quadrature-phase wavelet. Wavelets that have a common amplitude envelope are here defined as being a wavelet class. A wavelet class shares resolving power because the distribution of total energy is identical.

DISCUSSION

To determine total energy one must have two attributes of the wavelet signal: displacement (potential energy) and velocity (kinetic energy). These signals are orthogonal to each other. Only velocity is measured by the standard geophone. Total energy, the sum of kinetic and potential energy is proportional to the square of the amplitude envelope. The basic argument is that the instantaneous energy of a wavelet is not equal to zero simply because the wavelet amplitude is zero at that time instant. For steady-state oscillators, i.e., a pendulum, when the swing crosses the rest point and displacement (potential energy) is zero the velocity (kinetic energy) is maximum. Likewise, when the pendulum stops temporarily at its maximum swing, velocity (kinetic energy) is zero, but displacement (potential energy) is maximum. The total energy of a steady-state oscillator is constant although the magnitude of an attribute such as displacement or velocity is sinusoidal.

The signal of any analytic function can be represented as the sum of the output of steady-state oscillators (Fourier transform theory). Each of these components has a constant energy, but total instantaneous energy depends on the relative phase of the component signals. Thus a transient pulse (wavelet) can be represented by the sum of infinite sinusoids. When all of the frequency components are in-phase, energy is maximized. In general, this only occurs at one particular time instance over all time. The exceptions are when the signal is composed of harmonic functions, like a square wave. The phase alignment uniquely occurs for zero-phase wavelets and other in its class. For instance, both zero-phase and quadrature-phase have a maximum total instantaneous energy at $t=0$. The frequency components of the zero-phase wavelet are in-phase at this time instance with all phase angles being zero. The frequency components of the quadrature-phase wavelet are likewise in-phase, but the angle is $\pi/2$. Wavelets with a non-trival phase spectrum, i.e., minimum-phase, never have the situation where the frequency components are all in-phase. Thus, within a wavelet suite, only the zero-phase class wavelets have a time instant where the instantaneous total energy is equal to the maximum possible for the suite.

Wavelets of the zero-phase wavelet class have the narrowest possible energy distribution. By implication, they, thus, share maximum resolving power. This proof is accomplished by showing that within a wavelet suite the magnitude spectrum of the total energy envelope has the broadest possible energy magnitude spectrum for zero-phase class wavelets. Using the Scaling Property of Fourier transform theory (see, i.e., Brigham, 1974) this means that zero-phase class wavelets have the narrowest possible energy envelope. Because the proof depends on the amount of phase change in the phase spectrum, it is implied that a minimum-phase wavelet is the causal wavelet with maximum resolving power. The minimum-phase wavelet has the least possible dispersion, i.e., the least possible change of phase for causal wavelets.

CONCLUSIONS

Maximum resolving power is reserved for zero-phase wavelets and others of its class. It includes all wavelets with a constant or linear phase function. Although extant works (Berkhout, 1974 and 1984; Schoenberger, 1974; and Widess, 1984) clearly demonstrate that zero-phase wavelets have maximum resolving power, the methods used specifically exclude wavelets with other-but-constant phase spectra such as the quadrature-phase wavelet. A quadrature-phase wavelet has the same total energy distribution as a zero-phase wavelet and, therefore, has an identical resolving power. The extant methods would all be rectified if the evaluation was based on the wavelet envelope instead of the real component of the wavelet trace.

For a given wavelet suite, the zero-phase wavelet uniquely has the maximum possible amplitude. Therefore, on wiggle-trace seismic sections, the zero-phase wavelet has an advantage over all other wavelet possibilities including other wavelets of its class because it has the largest instantaneous amplitude. It might further be added that from a practical standpoint a 45-degree wavelet is probably not the one an interpreter would choose even if it has the same theoretical resolving power. That is a visual human factor, not a mathematical one. Energy related sections, such as reflection strength, are independent of constant-angle phase shift. Within a given wavelet class, all sections are identical. The same is true for instantaneous frequency. It is constant with phase shift. Instantaneous phase shows a rotation of phase colors with different phases, but the geometry of the section is constant.

REFERENCES

Berkhout, J.A., Related properties of minimum-phase and zero-phase time function: Geophysical Prospecting 22, 683-709.

_____ 1984, Seismic Resolution: Geophysical Press.

Brigham, E.O., 1974, The Fast Fourier Transform: Prentice-Hall.

Claerbout, J.F., 1985, Fundamentals of Geophysical Data Processing: Blackwell Scientific Publications.

Kallweit, R.S. and Wood, L.C., 1982, The limits of resolution of zero-phase wavelets: Geophysics 47, 1035-1046.

Knapp, R.W., 1990, Vertical resolution of thick beds, thin beds and thin-bed cyclothems: Geophysics 55, 1184-1191.

Robinson, E.A., 1964, The minimum-delay concept in system design, part III: Design Electronics, February.

Robinson, E.A., and Treitel, S., 1965, Dispersive digital filters: Reviews of Geophysics 3, 433-461.

_____ 1980, Geophysical Signal Analysis: Prentice-Hall.

Schoenberger, M., 1974, Resolution comparison of minimum-phase and zero-phase signals: Geophysics 39, 826-833.

Taner, M.T., Koelher, F., and Sheriff, R.E., 1979, Complex seismic trace analysis: Geophysics 44, 1041-1063.

Widess, M.B., 1982, Quantifying resolving power of seismic systems: Geophysics 47, 1160-1174.