DISCUSSION ON SOME PRACTICAL EQUATIONS WITH IMPLICATIONS TO HIGH-FREQUENCY SURFACE-WAVE TECHNIQUES

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Abstract

We discuss five useful equations related to high-frequency surface-wave techniques and their implications in practice. These equations are theoretical results from published literature regarding source selection, data-acquisition parameters, resolution of a dispersion curve image in the frequency-velocity domain, and the cut-off frequency of high modes. The first equation suggests Rayleigh waves appear in the shortest offset when a source is located on the ground surface, which supports our observations that surface impact sources are the best source for surface-wave techniques. The second and third equations, based on the layered earth model, reveal a relationship between the optimal nearest offset in Rayleigh-wave data acquisition and seismic setting—the observed maximum and minimum phase velocities, and the maximum wavelength. Comparison among data acquired with different offsets at one test site confirms the better data were acquired with the suggested optimal nearest offset. The fourth equation illustrates that resolution of a dispersion curve image at a given frequency is directly proportional to the product of a length of a geophone array and the frequency. We used real-world data to verify the fourth equation. The last equation shows that the cut-off frequency of high modes of Love waves for a two-layer model is determined by shear-wave velocities and the thickness of the top layer. We applied this equation to Rayleigh waves and multi-layer models with the average velocity and obtained encouraging results. This equation not only endows with a criterion to distinguish high modes from numerical artifacts but also provides a straightforward means to resolve the depth to the half space of a layered earth model.

Introduction

Shear (S)-wave velocities can be derived from inverting dispersive phase velocities of the surface (Rayleigh and/or Love) waves (e.g., Aki and Richards, 1980, p. 664). Near-surface S-wave velocities can be estimated by Spectral Analysis of Surface Waves (SASW) (Stokoe and Nazarian, 1983; Stokoe et al., 1989), which analyzes the dispersion curve of the ground roll to produce near-surface S-wave velocity profiles. The other method developed in the last a few years utilizes a multichannel recording system to estimate near-surface S-wave velocity from high-frequency (≥ 2 Hz) Rayleigh waves (Multichannel Analysis of Surface Waves—MASW, Park et al., 1999a; Xia et al., 1999). Errors associated with S-wave velocities obtained by MASW method are 15% or less and random (Xia et al., 2002a). Discussion of multi-channel analysis of surface-wave dispersion can also be found in Lin and Chang (2004).

There are more and more publications on utilizing surface waves in defining near-surface S-wave velocities (e.g., Xia et al., 2000a; 2003; Beaty et al., 2002; Beaty and Schmitt, 2003) and
In environmental studies, the MASW method was employed to generate 2-D shear-wave velocity fields calculated from inversion of Rayleigh-wave phase velocities and was successfully used to define bedrock interfaces and near-surface geological structures from 2 to 50 meters (Xia et al., 1998; Miller et al., 1999) and to determine a collapse feature in an extremely noisy environment (Xia et al., 2004a). The MASW method is one of the main components in a unified workflow for engineering seismology (Yilmaz and Eser, 2002). The MASW method was also associated with a fast and efficient method for geophone deployment—autojuggie (Steeples et al., 1999)—to increase efficiency in acquisition of surface wave data (Tian et al., 2003a and 2003b). The MASW was also applied in pavement testing (Ryden et al., 2004) and in shallow marine environment (Luke et al., 1996; Park et al., 2000). Inversion of the shallow seismic wavefield utilized all signals in a shot gather, including surface waves and body waves, was discussed by Forbriger (2003a; 2003b). Cross-correlation technique and Common Mid-Point (CMP) gather were used to generate dispersion image (Hayashi and Hikima, 2003). MASW applications in near-surface geophysics can be found in other numerous publications (e.g., Xia et al., 1997; 2002c; Ivanov et al., 2000; Park et al., 1996; 1999b; Calderon-Macias and Luke, 2002; and Lin et al., 2004). Accuracy and resolution of surface wave techniques were analyzed by Rix and Leipski (1991) using a numerical modeling method. Understanding the resolving power of MASW techniques and improving resolution of S-wave velocity results were discussed by Xia et al. (in view).

In this paper, we will discuss several equations in the published literature and their potential usage related to high-frequency Rayleigh wave data. The first three equations deal with acquiring high-quality surface-wave data. The fourth equation is associated with resolution of dispersion image in the frequency-velocity domain. The last equation, which provides a tool of cross-checking inverted results and fast estimating the depth to the half space, is related to higher mode data.

### Equations Related to Data Acquisition

With earthquake data, Nakano (1925) showed that the Rayleigh waves do not appear at places near the sources, i.e., where the epicentral distance \( x \) is smaller than

\[
x \leq \frac{c_R h}{\sqrt{V_p^2 - c_R^2}} \quad \text{or} \quad x \leq \frac{c_R h}{\sqrt{V_s^2 - c_R^2}},
\]

where \( h \) is the depth of source, and \( c_R, V_p, \) and \( V_s \) are velocities of Rayleigh waves, P-wave, and S-wave, respectively. It was proved that Rayleigh waves do not have their full amplitude near these limit distances. This is due to interferences with other kinds of waves (Ewing et al., 1957, p. 66). Equation (1) suggests a source type we should use in high-frequency Rayleigh wave survey is a ground surface impact source such as a vibrator (e.g., Xia et al., 1999), a weight dropper (e.g., Miller and Xia, 1999; Xia et al., 2002a), and a sledgehammer (e.g., Miller et al., 1999; Xia et al., 2002b). The surface impact source would generate Rayleigh waves in the shortest distance or the shortest time. Based on Equation (1), given a source on the ground surface, Rayleigh waves would be observed in a very closed offset.
In reality, Rayleigh waves may not be observed in a very closed offset. One of the critical parameters in acquisition of high-frequency surface wave data is the nearest offset (Park et al., 1999a; Xia et al., 2004b). How far away should the first geophone be located in a Rayleigh wave survey? The following discussion may answer this question. Given a two-layer model (one layer on the top of the half space), the minimum offset $x$ where the direct $S_v$-wave meets the reflected $P$-wave (the necessary condition for generating dispersive Rayleigh waves), so

$$x = \frac{2hV_s}{\sqrt{V_p^2 - V_s^2}} = \frac{2h}{\sqrt{(V_p/V_s)^2 - 1}},$$  \hspace{1cm} (2)

where $h$ is the depth to the first interface. It is common for near-surface materials that $V_p/V_s$ is in the range 2 – 5 so $x$ is in a range of 0.4$h$ to 1.2$h$. Assuming $V_p/V_s$ is 4, we obtain $x \approx h/2$ from Equation (2), which indicates Rayleigh waves would appear where the offsets are larger than one-half of the thickness of the first layer. As $V_p/V_s$ is reducing, the minimum offset for Rayleigh waves to occur could increase to the thickness of the first layer or even larger. When designing a Rayleigh wave survey, it is necessary to set the nearest offset larger than the minimum offset determined by Equation (2) if knowledge of $V_p/V_s$ and the thickness of the first layer $h$ are available.

Because longer wavelength components of Rayleigh waves require longer time or larger offsets to develop into plane waves, Equation (2) does not provide the optimum offset for the majority of surface-wave components. Zhang et al. (2004) proposed an optimum offset $A$ based on a layered earth model,

$$A = \frac{\lambda_{\text{max}} c_{R\text{min}}}{4\Delta c_R},$$  \hspace{1cm} (3)

where $\lambda_{\text{max}}$, $c_{R\text{min}}$, and $\Delta c_R$ are the longest wavelength, the minimum phase velocity of Rayleigh waves, and the difference between the maximum and minimum phase velocities, respectively. Zhang et al. (2004) also proposed the geophone spread length $C$ (the distance between the first geophone and the last geophone) would be the best when $C = 2A$.

We assessed Equation (3) with data acquired in Virginia Key, Florida, in March 2004. Surface-wave data were acquired using a 24-channel seismograph with 14-Hz vertical-component geophones that were deployed at 0.6-m intervals. The source was a 3.5-kg hammer vertically impacting a 0.3-m by 0.3-m metal plate. The first shot was acquired with the nearest offset of 4.5 m (Figure 1a). With the dispersion image (Figure 1b), we determined the longest wavelength $\lambda_{\text{max}} = 500$ m/s/25 Hz = 20 m, the minimum phase velocity $c_{R\text{min}} = 180$ m/s, and the difference between the maximum and minimum phase velocities $\Delta c_R = 500$ m/s – 180 m/s = 320 m/s. So the suggested optimum offset $A$ would be 2.8 m. Figure 2a shows the data acquired with the nearest offset of 3 m. Its dispersion image is shown in Figure 2b. Comparing Figures 1a and 2a, we noticed linearity of Rayleigh waves is much better with the data acquired with the nearest offset of 3 m (Figure 2a) than with the nearest offset of 4.5 m (Figure 1a), especially for data in a
rectangular window. In terms of surface-wave energy in the frequency-velocity ($f$-$v$) domain calculated by SurfSeis® developed by the Kansas Geological Survey (KGS) (Figures 1b and 2b), it is even clearer that the continuity (especially for frequencies $> 30$ Hz) and resolution (indicated by a double-end arrow around frequency of $28$ Hz) of the dispersive energy from the data with the nearest offset of $3$ m are better than that of the data with the nearest offset of $4.5$ m.

**Resolution of Dispersion Curve**

Modeling results (Park et al., 1998) show that resolution of the dispersion image in the $f$-$v$ domain will increase as the geophone spread length increases. Forbriger (2003a) provided an analytical result to assess resolution of dispersion image,

$$\Delta d = 1/ fC,$$

where $\Delta d$ is the half-width between the neighboring minima of dispersion energy in the $f$-$v$ domain, $f$ is the frequency, and $C$ is the geophone spread. We understand that resolution of the dispersion image could vary with algorithms that were used to generate dispersion image in the $f$-$v$ domain. Current comparison of several different algorithms can be found in Dal Moro et al. (2003). Our discussion will be limited to using the phase shift method (Park et al., 1998).

We used data acquired at the Fraser River delta, near Vancouver, British Columbia, Canada in 1998 to assess Equation (4). Complete discussion on the data can be found in Xia et al. (2000b). Multi-channel surface wave data were acquired using 4.5-Hz vertical geophones and a 60-channel Geometrics StrataView seismograph. Geophones were deployed at each of the eight sites at either a 0.6 or 1.2 m interval (depending on target depth range) with the nearest source-to-geophone offset in the range of 1.2 m to a farthest of 90 m. The geophone spread was placed as close as possible to the measurement well and always within $50$ m of it. Three to ten impacts were vertically stacked at each offset using an accelerated weight drop designed and built by the KGS. A record length of $2048$ milliseconds at a $1$-millisecond sample interval was selected for all sites. Data acquired at three wells were used to assess Equation (4).

The first data set was acquired at well FD92-11 (Figure 3a). The nearest offset was $18$ m with a geophone interval of $0.6$ m. Four pairs of results were generated with the data (Figure 3a). The first pair in Figures 3a and 3b was raw data in the $x$-$t$ domain and their dispersive energy in the $f$-$v$ domain, respectively. To estimate resolution of the dispersion image, we use a double-end arrow at frequencies $10$ Hz and $20$ Hz. The double-end arrow at $10$ Hz is twice as long as that at $20$ Hz. This agrees with Equation (4) that indicates resolution of dispersion image is one half at $10$ Hz compared with that at $20$ Hz. Even traces were used to generate the second pair (Figures 3c and 3d). The geophone spread $C$ of Figure 3c was the same as Figure 3a but its geophone interval was doubled. As indicated by Equation (4), resolution of dispersion image should remain the same. Both double-end arrows in Figures 3b and 3d at $10$ Hz were the same length, which also supports Equation (4) that resolution of dispersion image is determined by the geophone spread not the geophone interval.

We split the data in Figure 3a into two parts; one contains 30 traces nearer the source (Figure 3e) and their dispersion image (Figure 3f), and the other 30 traces further from the source (Figure 3g) and their dispersion image (Figure 3h). The geophone spread is the half of that used
in Figure 3a, but the geophone interval is the same as in Figure 3a. Based on Equation (4), we expect to see resolution reduce to one half \((\Delta d \text{ is doubled})\) as shown in Figure 3b. Both double-end arrows in Figures 3f and 3h at 10 Hz are only 50\% (not 100\%) longer than arrows in Figures 3b and 3d at 10 Hz, respectively.

The second data set was acquired at well FD97-2 (Figure 4a) and their dispersion image in the \(f-v\) domain is shown in Figure 4b. The 30 nearer tracers (Figure 4c) from the source were used to generate their dispersion image (Figure 4d) so the geophone spread is one half that used in Figure 4a but the geophone interval is the same as used in Figure 4a. The length of the double-end arrow in Figure 3d is 75\% (not 100\%) longer than that in Figure 4b.

The third data set was acquired at well FD97-7 (Figure 5a) and their dispersion image in the \(f-v\) domain (Figure 5b). The 30 nearer tracers from the source (Figure 5c) were used to generate their dispersion image (Figure 5d) so the geophone spread is one half as used in Figure 5a but the geophone interval is the same as used in Figure 5a. The length of the double-end arrow 3.7 Hz in Figure 5d is 50\% longer than that in Figure 5b.

As discussed in the previous examples, we may expect Equation (4) works well as frequency changes when the geophone spread is fixed in practice. On the other hand, if we double the geophone spread for a given frequency, resolution increase is normally less than 100\%.

### The Cut-off Frequency of Higher Modes

Higher mode data in Rayleigh wave survey are often observed at man-made structures such as dams and levees. They could be the sole source of information on S-wave velocities without drilling. Higher modes possess superb properties on penetration depth and resolution of inverted models in near-surface applications (Xia et al., 2000a, 2003, Beaty et al. 2002). The cut-off frequency \(f_n\) of the \(n\)th higher mode is the frequency that the \(n\)th higher modes exists only for \(f > f_n\). The cut-off frequency of the \(n\)th higher mode for Love wave is (Aki and Richards, 1980, p. 264)

\[
f_n = \frac{n\beta_1}{2h\sqrt{1 - \left(\frac{\beta_1}{\beta_2}\right)^2}},
\]

where \(\beta_1\) and \(\beta_2\) are S-wave velocities of the top and the half space (a two-layer model), respectively, \(h\) is the thickness of the top layer, and \(f_n\) is the cut-off frequency of the \(n\)th higher mode. For example, given a two-layer model with \(\beta_1 = 150\ \text{m/s}\) and \(\beta_2 = 250\ \text{m/s}\) \(h = 10\ \text{m}\) (Figure 6a), the first higher mode will occur when \(f_1 \geq 9.37\ \text{Hz}\), the second higher mode \(f_2 \geq 18.75\ \text{Hz}\), and the third higher mode \(28.11\ \text{Hz}\) (Figure 6b). By knowing the cut-off frequency, \(\beta_1\), and \(\beta_2\) (from the asymptotes at high and low frequencies), we can directly estimate the depth to the half space. Dispersion curves of the Rayleigh waves of the 2-layer model (Figure 6a) are also shown in Figure 6c. We may apply Equation (5) to Rayleigh wave data to estimate the depth to the half space based on the cut-off frequency \(f_1 (= 7\ \text{Hz})\), \(\beta_1 (= 150\ \text{m/s})\), and \(\beta_2 (= 250\ \text{m/s})\).
The estimated depth is 13.4 m. We should notice, for Rayleigh waves, the asymptote for higher modes at the high frequency is approaching \( \beta_1 (=150 \text{ m/s}) \) while the fundamental mode approaches \( 0.92*\beta_1 \) and the higher modes at the low frequency reach \( \beta_2 (= 250 \text{ m/s}) \) while the fundamental mode approaches \( 0.92*\beta_2 \).

There is no closed form of the cut-off frequency for Rayleigh waves for the two-layer model. However, Kennett (2001, p. 339-342) pointed out that the dispersion curves for Love and Rayleigh waves in the slowness-frequency domain are very similar at high frequencies \((f > 0.01 \text{ Hz})\). Modeling results showed that effects of P-wave velocity on higher modes are reduced to a negligible level compared with that on the fundamental mode (Xia et al., 2003). In the interval \( \beta_1 < c_R < \alpha_1 \) (\( \alpha_1 \) is P-wave velocity of the top layer), S-wave velocity is more dominant in higher Rayleigh modes, the character of which is akin to \( S_v \) Love modes (Kennett and Clarke, 1983). The high-frequency asymptote for phase velocities of higher Rayleigh modes is the S-wave velocity of the top layer (Kennett, 2001, p. 339), and the low-frequency component approaches the S-wave velocity of the half space for a two-layer model (Ewing et al., 1957, p. 194; also see Aki and Richards, 1980, p. 264). These properties are the same as Love waves. The cut-off frequencies for Rayleigh waves and Love waves are almost identical at least for the first four higher modes with a little phase shift (see Figures 16.8 and 16.9, Kennett, 2001).

By knowing the cut-off frequency \( f_n \), the high-frequency asymptotes, and the low frequency asymptotes, we are able to directly estimate the depth to the half space and distinguish higher modes from calculation artifacts in the \( f-v \) domain. Furthermore, we can use the Equation (5) to cross-check the inverted results. In the following discussion, we will apply Equation (5) to Rayleigh wave data and assess potential capability in cross-checking inverted models with modeling data. Equation (5) will also be applied to multi-layer models using the time-weighted average velocity \( \bar{v} = \left( \sum_{i=1}^{n-1} t_i v_i \right) / \left( \sum_{i=1}^{n-1} t_i \right) \), \( t_i = z_i/v_i \), where \( z_i \) is the thickness of the \( i \)th layer) to replace velocities on top of the half space.

A six-layer model (Xia et al., 1999) was used to test the feasibility of applying Equation (5) to six-layer Love and Rayleigh wave data (Figure 7). The depth to the half space is 12.8 m. The average S-wave velocity of the layer model (top five layers) is 348.6 m/s. The first three higher modes are shown in Figure 8. The cut-off frequencies of Love waves are: \( f_1 = 18 \text{ Hz}, f_2 = 33 \text{ Hz}, \) and \( f_3 = 49 \text{ Hz} \), and of Rayleigh waves: \( f_1 = 13 \text{ Hz}, f_2 = 22 \text{ Hz}, \) and \( f_3 = 37 \text{ Hz} \). The average cut-off frequency is 15.5 Hz for the Love waves (Figure 8a) and 12 Hz for the Rayleigh wave data (Figure 8b). The average cut-off frequency can be used to check the depth to the half space with the average velocity by Equation (5). In this case, we obtain 12.7 m with the average cut-off frequency of the Love wave data and 16.5 m with the average cut-off frequency of the Rayleigh wave data.

It is interesting to calculate an estimated depth to the half space directly from Equation (5) based on the high-frequency asymptote and the phase velocity at the lowest frequency of the first higher mode. For Love waves (Figure 8a), we use \( \beta_1 = 260 \text{ m/s}, \beta_2 = 729 \text{ m/s}, f_1 = 18 \text{ Hz}, \)
so the depth to the half space $h = 7.7$ m. For Rayleigh waves (Figure 8b), we use $\beta_1 = 260$ m/s, $\beta_2 = 702$ m/s, $f_1 = 13$ Hz, so the depth to the half space $h = 10.7$ m.

We also applied Equation (5) to a 14-layer model (Figure 9). The average S-wave velocity (top 13 layers) is 325.2 m/s. The depth to the half space is 13 m. The first three higher modes are shown in Figure 10. The cut-off frequencies of Love waves are: $f_1 = 16$ Hz, $f_2 = 31$ Hz, and $f_3 = 46$ Hz and the average is 15 Hz (Figure 10a), and of Rayleigh waves: $f_1 = 11$ Hz, $f_2 = 20$ Hz, and $f_3 = 35$ Hz and the average 12 Hz (Figure 10b). For this model, using the average velocity, we obtain 12.2 m with the average cut-off frequency of the Love wave data and 15.3 m with the average cut-off frequency of the Rayleigh wave data.

In the real world, if only the first higher mode is available, we would still be able to utilize the concept of the cut-off frequency in estimating an initial model and cross-checking inverted models or in finding an average velocity model. An estimated depth to the half space could directly calculated from Equation (5) based on the high frequency asymptote and the phase velocity at the lowest frequency of the first higher mode. For Love waves (Figure 10a), we use $\beta_1 = 230$ m/s, $\beta_2 = 700$ m/s, $f_1 = 16$ Hz, so the depth to the half space $h = 7.6$ m. For Rayleigh waves (Figure 10b), we use $\beta_1 = 250$ m/s, $\beta_2 = 660$ m/s, $f_1 = 11$ Hz, so the depth to the half space $h = 12.3$ m. From two multi-layer examples, it is interesting to notice that the direct calculation from Rayleigh waves based on the velocities determined by asymptotes provides better results than that from Love waves.

Conclusions

Five simple equations can be used as rule of thumb or guidelines in surface-wave surveys. The first equation suggests surface-impact sources were best source in surface-wave surveys. The second and third equations show the nearest offset is important in acquisition of surface wave data. Real data demonstrated the optimum offset may be determined by Equation (3). In practice, doubling the frequency can double resolution of the dispersion image in the $f$-$v$ domain as indicated by Equation (4), but doubling the geophone spread may not necessarily double resolution of dispersion image. The cut-off frequency equation provides a useful tool in estimating S-wave velocities of the top layer and the half space and the depth to the half space. Therefore, the cut-off frequency provides extra information that can be used for generating an initial model (a depth to the half space) and in cross-checking inverted models.

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Figure 1. (a) Raw Rayleigh wave data acquired at Virginia Key, Florida, with the nearest offset of 4.5 m. (b) Dispersion image of the data (a) in the $f$-$v$ domain. $\Delta d$ is a measure of resolution (sharpness) of dispersion image.

Figure 2. (a) Raw Rayleigh wave data acquired at Virginia Key, Florida, with the nearest offset of 3 m that is determined by Equation (3). (b) Dispersion image of the data (a) in the $f$-$v$ domain. The double-end arrow is 10% shorter than the one in Figure 1b.
Figure 3. (a) Raw Rayleigh wave data acquired at well FD92-11 (Xia et al., 2000b) and their dispersion image in the $f$-$\nu$ domain (b). The double-end arrow indicates (a) and (b) is a pair of results. (c) Only even traces of (a) used to generate their dispersion image (d) so the geophone spread is the same as (a) but the geophone interval is doubled. Both double-end arrows in (b) and (d) in 10 Hz are the same length. The double-end arrow at 20 Hz is one half of one at 10 Hz. (e) The 30 nearer traces from the source were used to generate their dispersion image (f) so the geophone spread is one half of that used in (a) but the geophone interval is the same as in (a). (g) The 30 further traces from the source were used to generate their dispersion image (h) so the geophone spread is one half of that used in (a) but the geophone interval is the same as in (a). Both double-end arrows in (f) and (h) are 50% longer than arrows in (b) and (d) at 10 Hz.
Figure 4. (a) Raw Rayleigh wave data acquired at well FD97-2 (Xia et al., 2000b) and their dispersion image in the f-v domain (b). The double-end arrow indicates (a) and (b) are a pair of results. (c) The 30 nearer tracers from the source in (a) were used to generate their dispersion image (d) so the geophone spread is one half of (a) but the geophone interval is the same as (a). The length of the double-end arrow at 10 Hz in (d) is 75% longer than that in (b).

Figure 5. (a) Raw Rayleigh wave data acquired at well FD97-7 (Xia et al., 2000b) and their dispersion image in the f-v domain (b). The double-end arrow indicates (a) and (b) are a pair of results. (c) The 30 nearer tracers from the source in (a) were used to generate their dispersion image (d) so the geophone spread is one half of (a) but the geophone interval is the same as (a). The length of the double-end arrow at 3.7 Hz in (d) is 50% longer than that in (b).
Figure 6. (a) A two-layer model. (b) Love wave response up to the third higher mode of a two-layer model. (c) Rayleigh wave response up to the third higher mode of the two-layer model.
Figure 7. A six-layer model (Xia et al., 1999).

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Figure 8. (a) Love wave response up to the third higher mode of the six-layer model in Figure 7. (b) Rayleigh wave response up to the third higher mode of the six-layer model in Figure 7.
A 14-layer model

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Figure 9. A 14-layer model.

Figure 10. (a) Love wave response up to the third higher mode of the 14-layer model in Figure 9. (b) Rayleigh wave response up to the third higher mode of the 14-layer model in Figure 9.