

Estimation of near-surface quality factors by inversion of Rayleigh-wave attenuation coefficients

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Summary

Quality factors of near-surface materials are as important as velocities of the materials in many applications. High-frequency (≥ 2 Hz) surface-wave data are generally inverted to determine near-surface shear (S)-wave velocities, in which only phase information of surface-wave data is utilized. Amplitude information of high-frequency surface-wave data can be used to determine quality factors of near-surface materials. Given S-wave velocity, compressional (P)-wave velocity, and Rayleigh-wave phase velocities, it is feasible to solve for S-wave quality factor Q_S and P-wave quality factor Q_P (for some specific velocity models) in a layered earth model down to 30 meters below the ground surface in many settings by inverting high-frequency Rayleigh-wave attenuation coefficients. Because an inversion system of this problem is unstable, we introduce a regularization parameter to limit a model length. Based on the linear nature of the inversion system, we search for a smooth model that is a trade-off solution between data (attenuation coefficient) misfit and model (quality factor) length. Several real-world examples demonstrate that the optimal regularization parameter can be found by the L -curve method and so can a smooth model.

Introduction

The most common measure of seismic-wave attenuation is the dimensionless quality factor Q or its inverse Q^{-1} (dissipation factor). As an intrinsic rock property, Q represents the ratio of stored to dissipated energy (Johnston and Toksöz, 1981). The quality factor as a function of depth is of fundamental interest in groundwater, engineering, and environmental studies, as well as in oil exploration and earthquake seismology. A desire to understand the attenuative properties of the earth is based on the observations that seismic-wave amplitudes are reduced as waves propagate through an elastic medium. This reduction is generally frequency dependent and, more importantly, attenuation characteristics can reveal unique information about lithology, physical state, and degree of rock saturation (Toksöz and Johnston, 1981). To fully understand seismic-wave propagation in the earth, the quality factors are parameters that must be known. Phase information of high-frequency Rayleigh waves is directly related to the shear (S)-wave velocity of near-surface materials and their amplitude information related to the quality factors

Laboratory experiments (Johnston et al., 1979) show that Q may be independent of frequency over a broad bandwidth ($10^2 - 10^7$ Hz), especially for some dry rocks. Q^{-1} in liquids, however, is proportional to frequency so that in some highly porous and permeable rocks Q^{-1} may contain a frequency-dependent component. This component may be negligible at seismic frequency, even in unconsolidated marine sediments (Johnston et al., 1979). Mitchell (1975) investigated Q structure of the upper crust in North America by inverting Rayleigh-wave attenuation coefficients in a layered earth model. In his work, Q was independent of frequency. Although some authors suggest that near-surface Q may be frequency dependent (Jeng et al., 1999), Xia et al. (2002) followed the laboratory results (Johnston et al., 1981) and Mitchell's work (1975) that Q is independent of frequency, allowing determination of Q as a function of depth based on amplitude attenuation of Rayleigh-wave data.

Modeling results that examined the relationship between Rayleigh-wave attenuation coefficients and compressional (P)-wave and S-wave quality factors (Q_P and Q_S) (Xia et al., 2002) suggested that it is feasible to invert attenuation coefficients of Rayleigh waves for quality factors. Modeling analysis also showed that Q_P may be inverted when V_S/V_P is greater than 0.45, a situation which is common in oil industry and crust seismology studies, and which also is not uncommon in near-surface materials. Modeling results also suggested that most contributions to Rayleigh-wave attenuation coefficients from Q_P are in a relatively higher frequency range while contributions from Q_S are in a lower frequency range. Using different weighting, therefore, on Q_P and Q_S in different frequency ranges may increase the possibility of obtaining Q_P . Modeling results (Xia et al., 2002), in addition, reveal that the stability of the inversion system for quality factors is much worse than that for S-wave velocities. Therefore, regularization and/or constraints on the quality factors is/are a must.

When no *a priori* information on the earth model is available, common practice is to seek a regularized solution that minimizes data misfit and the length of an earth model, for example, a regularized least-squares solution (Levenberg, 1944; Marquardt, 1963). The regularization parameter acts as a constraint on the model space (Tarantola, 1987) and can be determined by a trial-and-error method, especially because the number of unknowns is less than the number of observed data after discretizing an earth model (e.g., Xia et al., 1999, 2002, 2003, 2006, and in press on surface-wave inversion) or assuming a specific nonlayered earth model (Xia et al., 2006, on surface-wave inversion for a compressible Gibson half-space). A trade-off solution, equivalently, is a smooth solution calculated by a truncated inverse of a data kernel (Parker, 1994; Hansen, 1998) with the singular value decomposition (Lanczos, 1961; Golub and Reinsch, 1970). A constrained linear system was solved by introducing a regularization parameter for quality factors in a layered earth model (Xia et al., 2002). Herein, we use the L -curve method to determine an optimal regularization parameter in the inversion system.

Inversion of Rayleigh-wave attenuation coefficients

Basic equations

The relationship between Rayleigh-wave attenuation coefficients and the quality factors for P and S waves of a layered model were given by Anderson et al. (1965) as:

$$\alpha_R(f) = \frac{\pi f}{C_R^2(f)} \left[\sum_{i=1}^n P_i(f) Q_{Pi}^{-1} + \sum_{i=1}^n S_i(f) Q_{Si}^{-1} \right], \quad (1)$$

where $P_i(f) = V_{Pi} \frac{\partial C_R(f)}{\partial V_{Pi}}$, $S_i(f) = V_{Si} \frac{\partial C_R(f)}{\partial V_{Si}}$, $\alpha_R(f)$ is Rayleigh-wave attenuation coefficients in 1/length, f is frequency in Hz, Q_{Pi} and Q_{Si} are the quality factors for P and S waves of the i th layer, respectively; V_{Pi} and V_{Si} are the P-wave velocity and S-wave velocity of the i th layer, respectively; $C_R(f)$ is Rayleigh-wave phase velocity, and n is the number of layers of a layered earth model.

We adopted Kudo and Shima's work (1970) to calculate the attenuation coefficients, $A(x+dx, t) = A(x, t)e^{-\alpha dx}$, where A is Rayleigh-wave amplitude, α is a Rayleigh-wave attenuation coefficient, and x and dx are the nearest source-geophone offset and a geophone interval, respectively. After the Fourier transform with respect to time t , we obtain

$$\alpha_R(f) = - \frac{\ln \left[\frac{W(x+dx, f)}{W(x, f)} \sqrt{\frac{x+dx}{x}} \right]}{dx}, \quad (2)$$

where $\alpha_R(f)$ is the Rayleigh-wave attenuation coefficient as a function of frequency f , $W(f)$ is the amplitude at frequency f , and $\sqrt{\frac{x+dx}{x}}$ is a scaling factor in calculating the attenuation coefficient.

Inversion system

Equation 1 manifests the linear relationship between Rayleigh-wave attenuation coefficients and the dissipation factors for P and S waves (Q_P^{-1} and Q_S^{-1}). Theoretically, after determining S-wave velocities by inverting Rayleigh-wave phase velocities (Xia et al., 1999) and finding near-surface P-wave velocities by other seismic methods, such as reflection (Hunter et al., 1984; Steeples and Miller, 1990), refraction (Palmer, 1980), and/or tomography methods (Zhang and Toksöz, 1998; Ivanov et al., 2006), the dissipation factors (Q_P^{-1} and Q_S^{-1}) can be inverted directly for noise-free data using Equation 1. Because Equation 1 is a linear system, the same method used in Xia et al. (1999) can be employed directly to solve Q_S and/or Q_P from Rayleigh-wave attenuation coefficients. Equation 1 can be written as a matrix form

$$A\bar{X} = \bar{B} \quad (x_i > 0), \quad (3)$$

where \bar{X} is an inverse of quality factors (a model vector $1/Q$) with x_i as the i th component, \bar{B} is attenuation coefficients (a data vector $\alpha_R(f)$), and A is a data kernel matrix (Menke, 1984) determined by Equation 1. Menke (1984) discussed an algorithm to solve Equation 3. Equation 3 will provide accurate Q_P and Q_S if attenuation coefficients contain no error as a synthetic example showed (Xia et al., 2002). Solutions of Equation 3 are not guaranteed to exist or solutions may possess an unacceptable error when attenuation coefficients possess errors. Because of the instability of Equation 3, Xia et al. (2002) introduced a damping factor into the system. Our inversion problem can be described by the following system:

$$(A + \lambda I)\bar{X} = \bar{B} \quad (x_i > 0), \quad (4)$$

where I is the unitary matrix and λ is a damping factor. The question is how to determine the optimal damping factor.

L-curve method

The art of superposing regularization and/or constraints provides numerous opportunities to geophysicists who can solve geophysical inverse problems with their knowledge of the nature of a specific data set and an earth model. To reduce instability of geophysical inverse problems with some *a priori* information on the earth model (e.g., Li and Oldenburg, 1996, 2000), a regularized solution that minimizes data misfit and a length of difference between an earth model and a reference model is normally sought [Zhdanov, 2002; detailed discussion can also be found in Oldenburg and Li (2005)]. The regularization parameter with these inverse methods can be determined by Tikhonov's method or the *L*-curve method (Lawson and Hanson, 1974; Tikhonov and Arsenin, 1977; Hansen, 1992, 1998). In case of no *a priori* information available as we face in our real data, we seek a regularized solution that is a trade-off between data (attenuation coefficient) misfit and model (quality factor) length. Applications of the *L*-curve in inversion of geophysical data can be found in much of the literature (e.g., Hansen, 1992; Li and Oldenburg, 1999). Herein, an *L*-value is defined as

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$$L(\bar{X}, \lambda) = \left(\|A\bar{X} - \bar{B}\|_2 W \|A\bar{X} - \bar{B}\|_2 \right)^{1/2} + \lambda \|\bar{X}\|_2 = \phi_d + \lambda \phi_m, \tag{5}$$

where W is a weighting matrix that is normally determined by data accuracy, $\phi_d = \left(\|A\bar{X} - \bar{B}\|_2 W \|A\bar{X} - \bar{B}\|_2 \right)^{1/2}$ is the data misfit, and $\phi_m = \|\bar{X}\|_2$ is the model length. Because of the linear nature of the system (Equation 1), a plot of (ϕ_m, ϕ_d) normally shows the shape of an L -curve although interpretation of an L -curve might need some trained eyes.

A model corresponding to a small damping factor usually possesses large errors and results in a very long model length with a small data misfit because the model tries to represent errors in data (an example shown in Figure 1). On the other hand, a model corresponding to a large damping factor is usually stable but results in a large data misfit with a short model length (Figure 1). Trade-off models could be found in the trade-off zone on the plot of (ϕ_m, ϕ_d) . In dealing with real data, it may not be easy to determine the optimal damping factor from an L -curve. A practical way to determine the optimal damping factor is to plot $L(\lambda)$ as shown in the following examples.

Real-world examples

A shallow Rayleigh-wave survey was conducted in the Southwestern United States during the spring of 2010 to determine seismic properties of near-surface material. Rayleigh-wave data were acquired using a streamer that consists of 24 4.5-Hz vertical component geophones with a 1.2-m interval. The seismic source was a weight drop vertically impacting on a plate.

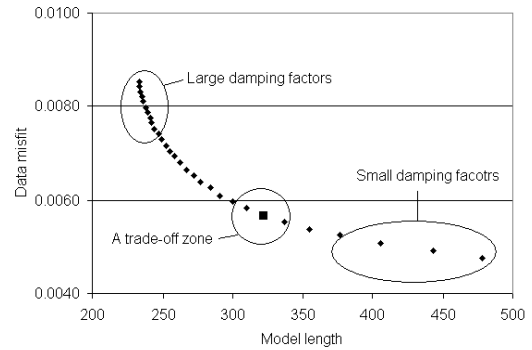


Figure 1. An example of an L -curve generated with different damping factors.

Although Rayleigh-wave data (Figure 2a) possess some noise, attenuation coefficients (triangles in Figure 2b) of this data set can be calculated by Equation 2. In solving for quality factors, we assumed $Q_p = 2Q_s$ in inversion. We found the smallest feasible damping factor was 0.018. With the damping factor changing up to 0.047 at an interval of 0.001, we found possible solutions (Figure 1) that produced a nearly perfect L -curve. For example, the first model on the right (Figure 1) associated with the smallest damping factor (0.018) possesses the longest model length (478) and the smallest data misfit (0.0048) and the first model on the left (Figure 1) associated with the largest damping factor (0.047) possesses the shortest model length (233) and the largest data misfit (0.0085). Models in the ranges at the both ends are not the best solution. We need to find a model in the trade-off zone (Figure 1). A plot of $L(\lambda)$ (Figure 2c) of this example can assist us to decide the optimal damping factor. The minimum point in the L -values suggested the best damping factor is 0.024 (a solid square in Figure 2c). The trade-off model associated with the best damping factor is indicated by a solid square in Figure 1, which produced a model with a model length of 322 and data misfit of 0.0057 (Figure 1). The attenuation coefficients calculated from the trade-off model are shown by solid squares in Figure 2b. Inverted S -wave quality factors (Q_s) are within a reasonable range

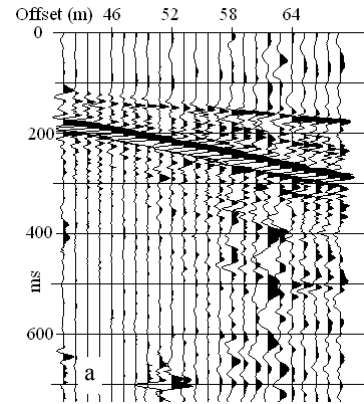


Figure 2. (a) Raw Rayleigh-wave data of the first example.

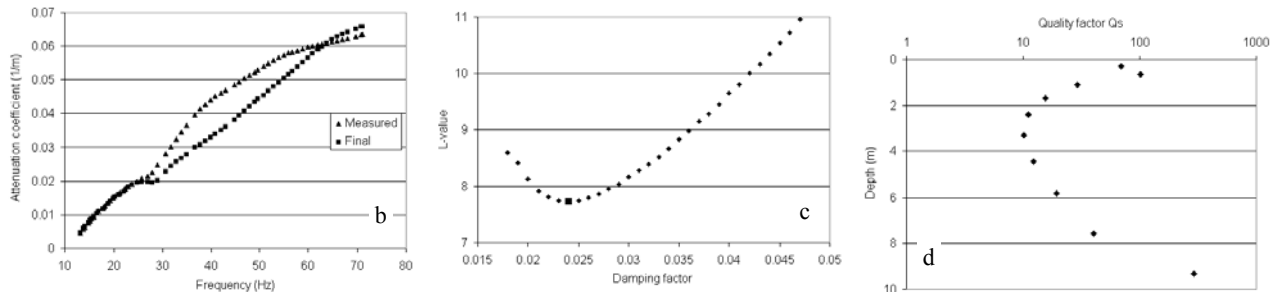


Figure 2. (b) Attenuation coefficients. Those labeled “Measured” were calculated by Equation 2 and those labeled “Final” were calculated by Equation 1 with inverted quality factors shown in Figure 2d. (c) L -value vs. damping factor. (d) An inverted Q_s model.

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except for the half space (Figure 2d). It is a little too high for near-surface materials possessing Q_S of nearly 300 at a depth of 10 m but it is not impossible.

Quality of Rayleigh-wave data of the second example (Figure 3a) is higher than the last example. We calculated attenuation coefficients (triangles in Figure 3b) using Equation 2. The same as we used in the last example, we assumed $Q_P = 2Q_S$ in inversion. We found the smallest feasible damping factor was 0.015. With the damping factor changing up to 0.044 at an interval of 0.001, we found possible solutions that produced a nearly perfect L -curve (Figure 3c). For example, the first model on the right (Figure 3c) associated with the smallest damping factor (0.015) possesses the longest model length (525) and the smallest data misfit (0.0043) and the first model on the left (Figure 3c) associated with the largest damping factor (0.044) possesses the shortest model length (232) and the largest data misfit (0.0066). A plot of $L(\lambda)$ suggested the trade-off model was associated with a damping factor of 0.02 (a solid square in Figure 3d), which produced a model with a model length of 309 and data misfit of 0.0048 (Figure 3c). The attenuation coefficients calculated from the trade-off model are shown by solid squares in Figure 3b. Inverted S -wave quality factors are within a reasonable range except for the half space (Figure 3e). It is the same as the last example that it is a little too high for near-surface materials possessing Q_S of nearly 300 at a depth of 7 m, but it is not impossible.

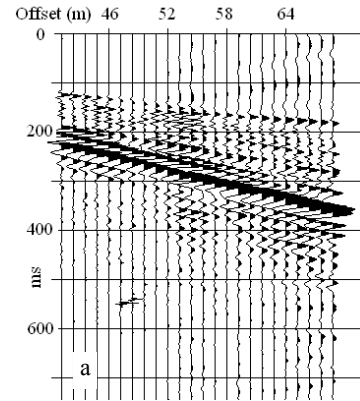


Figure 3. (a) Raw Rayleigh-wave data of the second example.

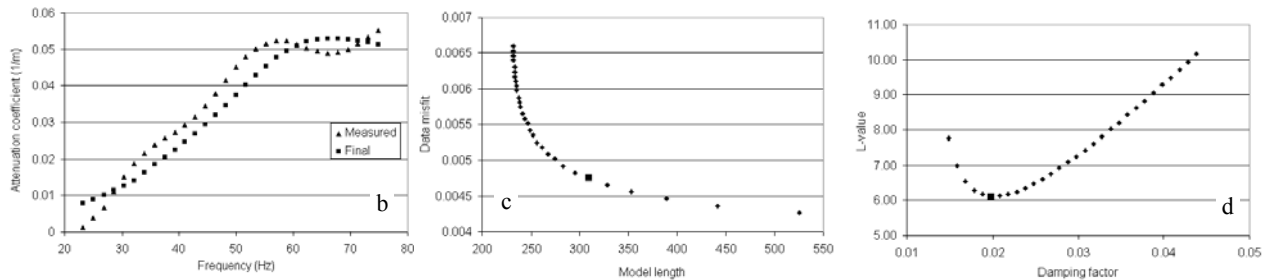


Figure 3. (b) Attenuation coefficients. Those labeled “Measured” were calculated using Equation 2 and those labeled “Final” were calculated using Equation 1 with inverted quality factors shown in Figure 3e. (c) An L -curve. (d) L -value vs. damping factor.

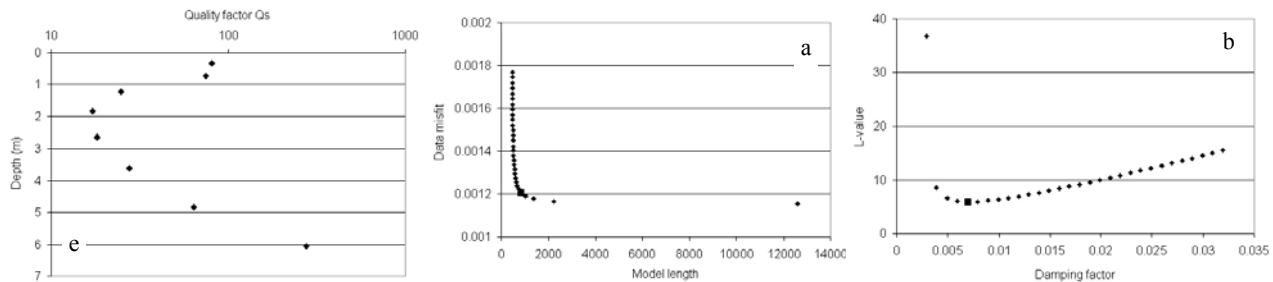


Figure 3. (e) An inverted Q_S model of the second example. Figure 4. (a) An L -curve and (b) L -value vs. damping factor of the third example. A solid square suggested a trade-off model.

We only show an L -curve and a plot of $L(\lambda)$ of the third example. The L -curve consisted of models with damping factors changing up to 0.032 at an interval of 0.001 from the smallest feasible damping factor of 0.003 (Figure 4a). Obviously a much smaller damping factor interval (< 0.001) should be used to find more models between the last two models on the right end of the L -curve. A plot of $L(\lambda)$ suggested the trade-off model was associated with a damping factor of 0.007 (a solid square in Figure 4b).

Conclusions

Real-world examples demonstrated that the L -curve method could provide a valuable measure in selection of feasible quality factor models in inverting high-frequency Rayleigh-wave attenuation coefficients. A trade-off quality-factor model could be determined by a plot of L -value vs. damping factor. Although the L -curve method works reasonably well for these data sets because the inversion system is linear, the proposed algorithm and the L -curve method may not work for some noisy data. This is the nature of the inversion system (Equation 4) as discussed by Xia et al. (2002).

EDITED REFERENCES

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REFERENCES

- Anderson, D. L., A. Ben-Menahem, and C. B. Archambeau, 1965, Attenuation of seismic energy in upper mantle : *Journal of Geophysical Research*, **70**, no. 6, 1441–1448, [doi:10.1029/JZ070i006p01441](https://doi.org/10.1029/JZ070i006p01441).
- Golub, G. H., and C. Reinsch, 1970, Singular value decomposition and least-squares solution: *Num. Math.* **14**, no. 5, 403–420, [doi:10.1007/BF02163027](https://doi.org/10.1007/BF02163027).
- Hansen, P. C., 1992, Analysis of discrete ill-posed problems by means of the L-curve: *Society of Industrial and Applied Mathematics Review*, **34**, 561–580.
- Hansen, P. C., 1998, Rank-deficient and discrete ill-posed problems, *Numerical aspects of linear inversion*: Society of Industrial and Applied Mathematics.
- Hunter, J. A., S. E. Pullan, R. A. Burns, R. M. Gagne, and R. L. Good, 1984, Shallow seismic reflection mapping of the overburden-bedrock interface with the engineering seismograph--Some simple techniques: *Geophysics*, **49**, 1381–1385, [doi:10.1190/1.1441766](https://doi.org/10.1190/1.1441766).
- Ivanov, J., R. D. Miller, J. Xia, D. W. Steeples, and C. B. Park, 2006, Joint analysis of refractions with surface waves: An inverse solution to the refraction-traveltime problem: *Geophysics*, **71**, no. 6, R131–R138, [doi:10.1190/1.2360226](https://doi.org/10.1190/1.2360226).
- Jeng, Y., J. Tsai, and S. Chen, 1999, An improved method of determining near-surface Q: *Geophysics*, **64**, 1608–1617, [doi:10.1190/1.1444665](https://doi.org/10.1190/1.1444665).
- Johnston, D. H., 1981, Attenuation: A state-of-the-art summary: *in* M. N. Toksöz, and D. H. Johnston, ed., *Seismic Wave Attenuation*: SEG, 123-135.
- Johnston, D. H., and M. N. Toksöz, 1981, Definitions and terminology, *in* M. N. Toksöz, and D. H. Johnston, ed., *Seismic Wave Attenuation*: SEG, 1-5.
- Johnston, D. H., M. N. Toksöz, and A. Timur, 1979, Attenuation of seismic waves in dry and saturated rocks: II. Mechanisms: *Geophysics*, **44**, 691–711, [doi:10.1190/1.1440970](https://doi.org/10.1190/1.1440970).
- Kudo, K., and E. Shima, 1970, Attenuation of shear wave in soil: *Bulletin of the Earthquake Research Institute*, **48**, 145–158.
- Lawson, C. L., and R. J. Hanson, 1974, *Solving least-squares problems*: Prentice-Hall, Inc.
- Lanczos, C., 1961, *Linear differential operators*: D. van Nostrand Co.
- Levenberg, K., 1944, A method for the solution of certain nonlinear problems in least squares: *Quarterly of Applied Mathematics*, **2**, 164–168.
- Li, Y., and D. W. Oldenburg, 1996, 3-D inversion of magnetic data: *Geophysics*, **61**, 394–408, [doi:10.1190/1.1443968](https://doi.org/10.1190/1.1443968).
- Li, Y., and D. W. Oldenburg, 1999, 3D inversion of DC resistivity data using an L-curve criterion: 69th Annual International Meeting, SEG, Expanded Abstracts, 251-254.
- Li, Y., and D. W. Oldenburg, 2000, Incorporation geological dip information into geophysical inversion: *Geophysics*, **65**, 148–157, [doi:10.1190/1.1444705](https://doi.org/10.1190/1.1444705).

- Marquardt, D. W., 1965, An algorithm for least squares estimation of nonlinear parameters: *Journal of the Society for Industrial and Applied Mathematics*, **2**, 431–441.
- Menke, W., 1984, *Geophysical data analysis--Discrete inversion theory*: Academic Press, Inc.
- Mitchell, B. J., 1975, Regional Rayleigh wave attenuation in North America: *Journal of Geophysical Research*, **80**, no. 35, 4904–4916, [doi:10.1029/JB080i035p04904](https://doi.org/10.1029/JB080i035p04904).
- Oldenburg, D. W., and Y. Li, 2005, Inversion for geophysics, A tutorial, *in* D. K. Butler, ed., *Near-Surface Geophysics*: SEG, 89-150.
- Palmer, D., 1980, The generalized reciprocal method of seismic refraction interpretation: SEG.
- Parker, R. L., 1994, *Geophysical inverse theory*: Princeton University Press.
- Steeple, D. W., and R. D. Miller, 1990, Seismic-reflection methods applied to engineering, environmental, and ground-water problems, *in* S. H. Ward, ed., *Geotechnical and Environmental Geophysics 1*: SEG, 1-30.
- Tarantola, A., 1987, *Inverse problem theory*: Elsevier Science B.V.
- Tikhonov, A. N., and V. Y. Arsenin, 1977, *Solution of ill-posed problems*: W.H. Winston and Sons.
- Toksöz, M. N., and D. H. Johnston, 1981, Preface, *in* M. N. Toksöz, and D. H. Johnston, ed., *Seismic Wave Attenuation*, SEG, v - vi.
- Xia, J., R. D. Miller, and C. B. Park, 1999, Estimation of near-surface shear-wave velocity by inversion of Rayleigh wave: *Geophysics*, **64**, 691–700, [doi:10.1190/1.1444578](https://doi.org/10.1190/1.1444578).
- Xia, J., R. D. Miller, C. B. Park, and G. Tian, 2002, Determining Q of near-surface materials from Rayleigh waves: *Journal of Applied Geophysics*, **51**, no. 2-4, 121–129, [doi:10.1016/S0926-9851\(02\)00228-8](https://doi.org/10.1016/S0926-9851(02)00228-8).
- Xia, J., R. D. Miller, C. B. Park, and G. Tian, 2003, Inversion of high frequency surface waves with fundamental and higher modes: *Journal of Applied Geophysics*, **52**, no. 1, 45–57, [doi:10.1016/S0926-9851\(02\)00239-2](https://doi.org/10.1016/S0926-9851(02)00239-2).
- Xia, J., Y. Xu, R. D. Miller, and C. Chen, 2006, Estimation of elastic moduli in a compressible Gibson half-space by inverting Rayleigh wave phase velocity: *Surveys in Geophysics*, **27**, no. 1, 1–17, [doi:10.1007/s10712-005-7261-3](https://doi.org/10.1007/s10712-005-7261-3).
- Xia, J., Y. Xu, R. D. Miller, and C. Zeng, A trade-off solution between model resolution and covariance in surface-wave inversion: *Pure and Applied Geophysics*, in press.
- Zhang, J., and M. N. Toksöz, 1998, Nonlinear refraction travelttime tomography: *Geophysics*, **63**, 1726–1737, [doi:10.1190/1.1444468](https://doi.org/10.1190/1.1444468).
- Zhdanov, M. S., 2002, *Geophysical inverse theory and regularization problems*: Elsevier Science B.V.