Comparative analysis on sensitivities of Love and Rayleigh waves

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Summary

Love waves are generally less employed than Rayleigh waves in near-surface shear-wave (S-wave) investigations because acquiring S-wave data is not as easy as acquiring P-wave data. The dispersion of Love waves, however, is independent of Poisson's ratio. This character can be the valuable basis to improve the S-wave velocity imaging. The sensitivity of earth properties to the dispersion curve of surface waves (Love and Rayleigh waves) is fundamental to determining the accuracy of an S-wave velocity model. By analyzing sensitivities to the dispersion curves of Love and Rayleigh waves comparatively, modes and frequency ranges of surface-wave dispersion data can be chosen appropriately during the inverse procedure, which helps to improve the accuracy of S-wave velocity profiles. For layered earth models, the sensitivities are defined by the percentage changes instead of partial derivatives. Analysis based on a regular model and an irregular model shows that, for earth properties such as the S-wave velocity and the thickness of each layer, Love waves are more sensitive than Rayleigh waves in most frequency ranges. Although the low velocity layer can dramatically influence the sensitivities of both Love and Rayleigh waves, the sensitivity of Love-wave to changes in S-wave velocity is much higher at low frequencies. Furthermore, Love waves are more sensitive than Rayleigh waves to the change of S-wave velocity in half-space. That means Love waves can recovery the S-wave velocity of the half-space more easily. All these indicate that the joint inversion of multimode Love waves with Rayleigh waves would obtain a more accurate S-wave profile.

Introduction

The S-wave velocity of near-surface materials is one of the most important parameters in many environmental and engineering geophysical studies. Since the late 1990s the surface-wave methods such as the multichannel analysis of surface waves (MASW) (McMechan and Yedlin, 1981; Song et al. 1989; Park et al., 1999) have been widely utilized to obtain the S-wave velocity profiles in near-surface geophysics (e.g., Xia et al., 1999). As a particular type of surface waves, Rayleigh-wave has received much attention in recent years. On the other side, as another type of surface waves, Love-wave is seldom employed. In some cases, it may be a menace to shallow S-wave reflection surveying (Miller et al., 2001). However, Love waves can also provide valuable information in engineering studies to improve the S-wave velocity imaging (Safani et al., 2005).

Love waves and Rayleigh waves travel with orthogonal polarization along the same propagation path. Rayleigh waves are generated by the interfering of P- and SV-waves. Differ from that, Love waves are the result of total internal reflection of multiple SH-waves. And the dispersion of Love waves is independent of Poisson's ratio (Aki and Richards, 1980). This character of Love-wave is very useful when other information is unavailable (e.g. borehole or P-wave data), which could be a pitfall in Rayleigh-wave inversion. Actually, simultaneous inversion of phase velocity of Love and Rayleigh waves has been proved as an effective method in the study of the earth structures (Lee and Solomon, 1979).

The sensitivity of earth properties to the dispersion curve of surface waves (Love and Rayleigh waves) is fundamental to determining the accuracy of an S-wave velocity model. For linear inversion such as the Levenberg-Marquardt (L-M) method (Levenberg, 1944; Marquardt, 1963), the analytic partial derivative is a key in determining modifications to model parameters and affects the convergence of inverse procedure dramatically. Analysis of the Jacobian matrix by Xia et al. (1999, 2003) demonstrates that the change in phase velocity of surface waves is dominantly influenced by the S-wave velocity. The fundamental mode of both Love and Rayleigh waves can be easily inverted for S-wave velocities. And higher modes are more sensitive to the S-wave velocity than the fundamental mode. Higher mode data can also be used to stabilize the inverse procedure and increase the resolution of the inverted S-wave velocities (Xia et al., 2003). Numerical studies by Feng et al. (2005) confirm that the sensitivities of different modes are concentrate in different frequency bands. Furthermore, analysis of the partial derivatives of surface-wave phase velocity by the method of Lai and Rix (1998) shows that the sensitivity and inversion stability of Love waves are higher than those of Rayleigh waves for a same model (Safani et al., 2005).

By analyzing the sensitivities to the dispersion curves of Love and Rayleigh waves comparatively, modes and frequency ranges of surface-wave dispersion data can be chosen appropriately during the inverse procedure, which helps to improve the accuracy of S-wave velocity profiles.

Sensitivities

The phase velocity sensitivities were defined according to Feng et al. (2001) as follows:

$$S_{Vi} = \frac{1}{c(f)} \frac{\partial c(f)}{\partial V_{si}} \delta V_{si} \approx \frac{c(f, V_{si} + \alpha \cdot V_{si}) - c(f, V_{si})}{c(f, V_{si})} \cdot 100\%$$

$$S_{Hi} = \frac{1}{c(f)} \frac{\partial c(f)}{\partial H_i} \delta H_i \approx \frac{c(f, H_i + \beta \cdot H_i) - c(f, H_i)}{c(f, H_i)} \cdot 100\%$$

where c is the phase velocity (m/s), f is the frequency (Hz), V_{si} and H_i are the S-wave velocity (m/s) and thickness (m) of layer i, respectively, and α and β are perturbation factors applied to the model parameters. S_{Vi} and S_{Hi} are the sensitivities of phase velocity with respect to S-wave velocity and thickness of layer i, respectively. Instead of partial derivatives, the sensitivities are defined as the percentage changes. Here α and β are both given the value of a 25%, and the sensitivity is the percentage change of phase velocity due to a 25% change in an S-wave velocity (or thickness).

The dispersion functions of Love and Rayleigh waves can be calculated by Knopoff's method (Schwab and Knopoff, 1972). This efficient approach can reduce the computer time of forward modeling. For each layer of the model, the sensitivities of different modes can be calculated by changing the corresponding model parameters (such as the S-wave velocity or the thickness).

Two models were used to test the sensitivities of Love waves and Rayleigh waves. One is a two-layer regular model (Xia et al., 1999). Another is a five-layer irregular model with a thin low velocity layer (LVL) underlain by a large elastic contrast (Safani et al., 2006).

Figure 1 shows the "true" S-wave profiles of the two models. And Figure 2 shows the corresponding dispersion curves of Love and Rayleigh waves. For the regular model (model 1), the dispersion curves were calculated over frequency 1-100 Hz, at a 1 Hz interval. For the irregular model (model 2) with a LVL, the frequency range is 1-120 Hz, at a 1 Hz interval.

Comparative analysis

From the sensitivity curves of each layer of the two models delineated in Figures 3 and 4, it is easy to find that for both Love and Rayleigh waves, higher sensitivities of different modes occur in the different frequency ranges. The higher sensitive frequency bands of Love waves are much wider than those of Rayleigh waves. For most modes of each layer, Love waves differ from Rayleigh waves in the sensitive frequency bands. For the fundamental mode, Love waves are more sensitive than Rayleigh waves to the S-wave velocity at the low frequency range. This may interpret why Love waves can recovery the S-wave velocity of half-space more accurately than Rayleigh waves. The absolute values of sensitivities to S-wave velocities of Love waves are much higher than those of Rayleigh waves. In another word, Love waves are more sensitive than Rayleigh waves to changes in S-wave velocity. Conclude on this, the joint inversion of Love waves and Rayleigh waves may obtain a more accurate S-wave profile.

The LVL can influence the sensitivities of both Love and Rayleigh waves dramatically. The sensitivities of the LVL are much higher than those of the other layers due to the S-wave velocity contrast. That means the LVL can be easily defined in multimode inversion. On the other hand, the sensitivities of the layers under the LVL are very low, which implies that the inversion results for layers that are under the LVL may possess higher uncertainty.

For the half-space, Love waves are more sensitive than Rayleigh waves to changes in S-wave velocity. In both fundamental and higher modes, Love waves have wider sensitive frequency bands. This may because the invariably longer wavelengths (higher phase velocities) of Love waves at low frequencies (Safani et al., 2005).

The sensitivity curves to the thickness (Figures 3 and 4) demonstrate that Love waves are less sensitive at low frequency but more sensitive at high frequency to changes in thickness than Rayleigh waves. In the higher modes, this phenomenon is particularly conspicuous. However, the high sensitivity frequency bands of Love waves are wider than those of Rayleigh waves, which indicates that higher modes of Love waves dispersion data can be the valuable information during the inversion procedure.

Conclusions

The dispersion of Love waves is independent of Poisson's ratio. For earth properties such as the S-wave velocity and the thickness of each layer, Love waves are more sensitive than Rayleigh waves in most frequency ranges. The low frequency Love waves have the high sensitivities to the S-wave velocity. Although the LVL can dramatically influence the sensitivities of both Love and Rayleigh waves, the sensitive frequency bands of Loves waves are wider than those of Rayleigh waves. For the half-space of either regular or irregular models, Love waves are more sensitive to changes in S-wave velocity than Rayleigh waves, which means Love waves can easily recovery the S-wave velocity of the half-space. And the joint inversion of multimode Love waves with Rayleigh waves would obtain the S-wave profiles more accurately.

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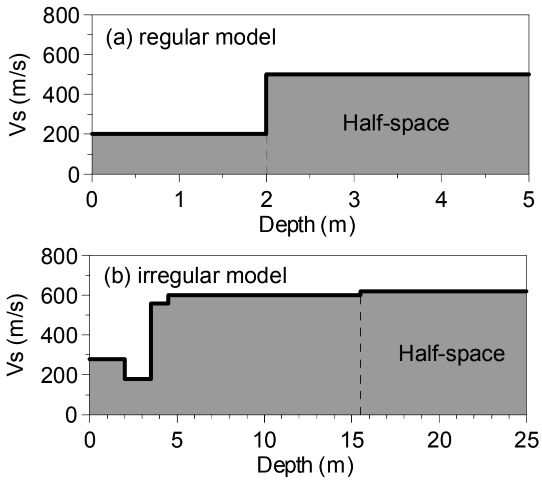

Figure 1: The shear-wave velocity profiles of the layered earth models: (a) regular model; (b) irregular model.

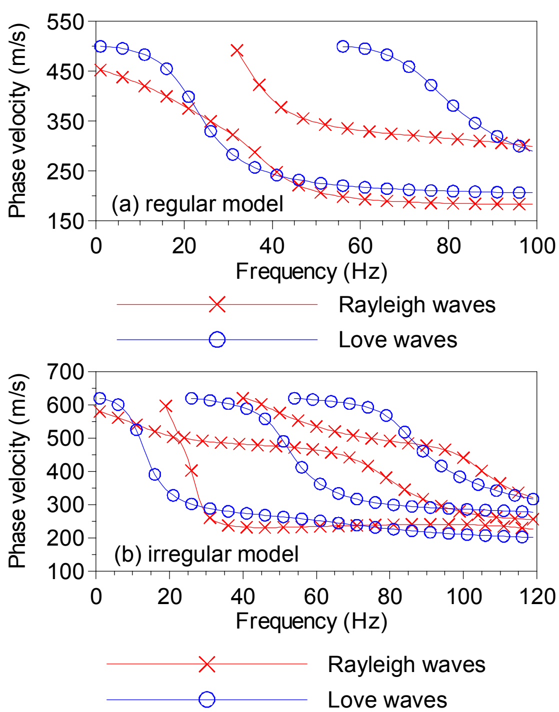

Figure 2: The dispersion curves of Love and Rayleigh waves of the two model. The lines with circle represent the dispersion curves of Love waves, and the lines with cross represent the dispersion curves of Rayleigh waves: (a) the two-layer regular model; (b) the irregular model with a LVL in the second layer.

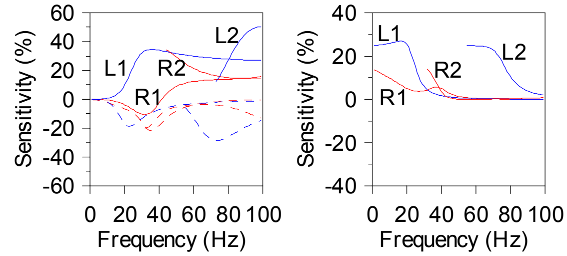

Figure 3: The sensitivity curves of Love and Rayleigh waves for the regular model (model 1). The solid lines are the sensitivities to the shear-wave velocity, and the dash lines are the sensitivities to the thickness of each layer. Label R1, R2, L1, L2 represent the sensitivity of fundamental mode Rayleigh waves, the 1st higher mode Rayleigh waves, the fundamental mode Love waves, the 1st higher model Love waves, respectively: (a) the first layer; (b) the half-space.

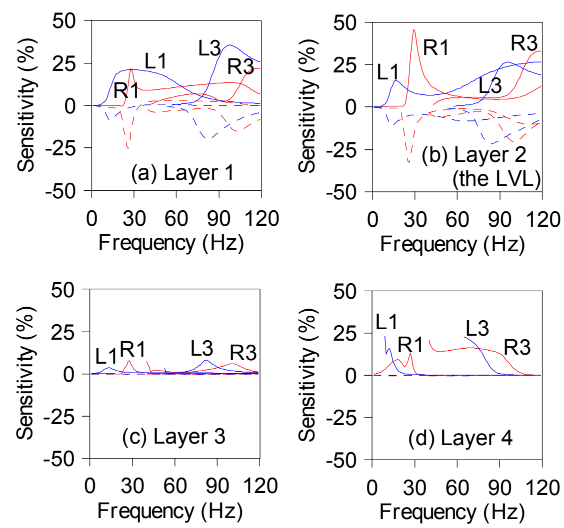

Figure 4: The sensitivity curves of Love and Rayleigh waves for the irregular model (model 2). The solid lines are the sensitivities to the shear-wave velocity, and the dash lines are the sensitivities to the thickness of each layer. Label R1, R3, L1, L3 represent the sensitivity of fundamental mode Rayleigh waves, the 2nd higher mode Rayleigh waves, the fundamental mode Love waves, the 2nd higher model Love waves and so on, respectively: (a) sensitivities of layer 1; (b) sensitivities of layer 2 (the LVL); (c) sensitivities of layer 3; (d) sensitivities of layer 4 (the layer nearest to the half-space).

EDITED REFERENCES

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