Sensitivity of high-frequency Rayleigh-wave data revisited

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Summary

Rayleigh-wave phase velocity of a layered earth model is a function of frequency and four groups of earth properties: P-wave velocity, S-wave velocity (Vs), density, and thickness of layers. Analysis of the Jacobian matrix (or the difference method) provides a measure of dispersion curve sensitivity to earth properties. Vs is the dominant influence for the fundamental mode (Xia et al., 1999) and higher modes (Xia et al., 2003) of dispersion curves in a high frequency range (>2 Hz) followed by layer thickness. These characteristics are the foundation of determining S-wave velocities by inversion of Rayleigh-wave data. More applications of surface-wave techniques show an anomalous velocity layer such as a high-velocity layer (HVL) or a low-velocity layer (LVL) commonly exists in near-surface materials. Spatial location (depth) of an anomalous layer is usually the most important information that surface-wave techniques are asked to provide. Understanding and correctly defining the sensitivity of high-frequency Rayleigh-wave data due to depth of an anomalous velocity layer are crucial in applying surface-wave techniques to obtain a Vs profile and/or determine the depth of an anomalous layer. Because depth is not a direct earth property of a layered model, changes in depth will result in changes in other properties. Modeling results show that sensitivity at a given depth calculated by the difference method is dependent on the Vs difference (contrast) between an anomalous layer and surrounding layers. The larger the contrast is, the higher the sensitivity due to depth of the layer. Therefore, the Vs contrast is a dominant contributor to sensitivity of Rayleigh-wave data due to depth of an anomalous layer. Modeling results also suggest that the most sensitive depth for an HVL is at about the middle of the depth to the half-space, but for an LVL it is near the ground surface.

Introduction

Elastic properties of near-surface materials and their effects on seismic wave propagation are of fundamental interest in groundwater, engineering, and environmental studies. S-wave velocity (Vs) is one of the key parameters in construction and earthquake engineering (e.g., Yilmaz and Eser, 2002; Yilmaz et al., 2006). Vs can be derived from inverting dispersive phase velocities of the surface (Rayleigh and/or Love) waves (e.g., Dorman and Ewing, 1962). Inversion of high-frequency Rayleigh-wave data (Xia et al., 1999) can provide near-surface S-wave velocities with random errors of 15% or less after comparison with direct borehole measurements (Xia et al., 2000, 2002a, 2002b, 2002c, and 2006) and be applied to near-surface geological problems (Miller et al., 1999; Tian et al., 2003a and 2003b; Xia et al., 2004; Ivanov et al., 2006a and 2006b). If higher-mode data are available, the accuracy of an inverted S-wave velocity can be significantly improved (Xia et al., 2003; Beaty et al., 2002; Beaty and Schmitt, 2003; Luo et al., in press; Song et al., in press). More publications have appeared on utilizing surface waves in defining an anomalous layer such as nonuniqueness in inversion of Rayleigh-wave data for shallow profiles containing a high-velocity layer (HVL) (Calderón-Macías and Luke, 2007), sensitivity due to an HVL (Jin et al., 2007), and inversion of multimode Rayleigh wave dispersion curves due to models with a low velocity layer (LVL) (Liang, et al., in review).

Sensitivity analysis of Rayleigh-wave data is a key to understanding the resolution power of Rayleigh-wave data and provides guideline for application of surface wave techniques in practice. The Jacobian matrix, which was calculated by numerical partial derivatives and was verified by variational techniques (Aki and Richards, 1980), provides a measure of dispersion curve sensitivity to earth properties (Xia et al., 1999). When S-wave velocities increase gradually with depth (Table 1 from Xia et al., 1999), increasing Vs by 25% in the model results in an average relative change of 39% in Rayleigh-wave phase velocity. Vs are the dominant influence for the fundamental mode. For the same model, higher modes are even more sensitive to Vs because higher modes are much less sensitive to other properties in comparison to the fundamental mode (Xia et al., 2003).

Table 1. A layered earth model parameters (Xia et al., 1999).

Layer number	**Vs** (m/s)	**Vp** (m/s)	ρ (g/cm^3)	**H** (m)
1	194.0	650.0	1.82	2.00
2	270.0	750.0	1.86	2.30
3	367.0	1400.0	1.91	2.50
4	485.0	1800.0	1.96	2.80
5	603.0	2150.0	2.02	3.20
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

When studying sensitivity of a model with an anomalous layer (HVL or LVL), people are most interested in sensitivity of Rayleigh-wave data due to depth of the layer. Because depth is not a direct earth property of the layered model, changes in depth of a layer will result in changes in other properties. Thus, sensitivity (dS) calculated by the difference method (dS=(f(h+dh)-f(h))/f(h), where h is depth to an anomalous layer) is affected by the Vs difference (contrast) between an anomalous layer and surrounding layers. Following modeling results show sensitivity of Rayleigh-wave data due to depth of an anomalous layer is mainly dependent on the Vs contrast.

Sensitivity of Rayleigh-wave data due to depth of layer

It is well known that Rayleigh-wave phase velocity of a layered earth model is a function of frequency and four groups of earth properties: P-wave velocity (Vp), S-wave velocity (Vs), density (ρ), and thickness of layers (**H**). So the difference method can be directly used to calculate sensitivity of Rayleigh-wave data with respect to these properties (Xia et al., 1999 and 2003). When considering sensitivity of Rayleigh-wave data due to depth of layer, sensitivity calculated by the difference method is affected by the Vs contrast because changes in depth cause changes in other properties. For example, if depth of layer 3 (Table 1) increases by 10 percent, the total thickness of the upper two layers will increase by 10% and the total thickness of layers 4 and 5 will decrease by 10%. A possible model could be one shown in Table 2. Because the thickness of surrounding layers are changed during sensitivity calculation by the difference method, the Vs contrast between an anomalous layer and surrounding layers will play an important role in sensitivity analysis (Xia et al., 2005). Obviously sensitivity of Rayleigh-wave data due to depth of layer 3 is dependent on the Vs contrast if the difference method is used with the Rayleigh-wave phase velocities due to models in Tables 1 and 2. As an extreme case, if Vs of layers 2, 3, and 4 (Table 2) are the same, the sensitivity of Rayleigh-wave data due to depth of layer 3 is zero.

Table 2. A 10% change in depth of layer 3 of the model in Table 1.

Layer number	**Vs** (m/s)	**Vp** (m/s)	ρ (g/cm^3)	**H** (m)
1	194.0	650.0	1.82	2.00
2	270.0	750.0	1.86	2.73
3	367.0	1400.0	1.91	2.50
4	485.0	1800.0	1.96	2.37
5	603.0	2150.0	2.02	3.20
the half space	740.0	2800.0	2.09	infinite

Modeling results of an HVL

An HVL (Vs = 400 m/s) is in a homogeneous half-space (Vs = 200 m/s) (Figure 1). The homogeneous half-space consists of 40 layers with a thickness of 0.2 m each on the top of the half-space. Thus, the depth to the half-space is 8 m. Vp and density are the same for the HVL and the homogeneous half-space: Vp = 1300 m/s and ρ = 1.75 g/cm^3. The depth to the center of the HVL changes from 0.5 to 6.6 m and the thickness of the HVL from 1 to 6 m. The maximum relative change due to a fixed depth change (dh = 0.2 m) is shown in Figure 2. The results show that the maximum sensitivity is around 30% for the 2-m thick HVL at depth of 3.4 m (a solid dot) and the most sensitive depth is around 3 m for the homogeneous background model.

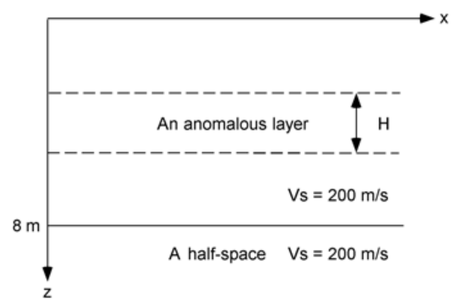

Figure 1. An anomalous layer is within a homogeneous half-space.

To analyze the role of the Vs contrast, we increased the Vs of the HVL to 800 m/s and Vp to 1403 m/s (a Poisson solid) and kept densities the same. Sensitivity results for this model (Figure 3) show that the general pattern of the sensitivity is almost the same as Figure 2 except that the maximum sensitivity with thickness of 2 m at depth of 3.4 m is doubled (60% as indicated by a solid dot) compared with the same case in Figure 2. Sensitivity is almost doubled at each depth for a 2-m thick HVL (Figure 4). We notice that the larger

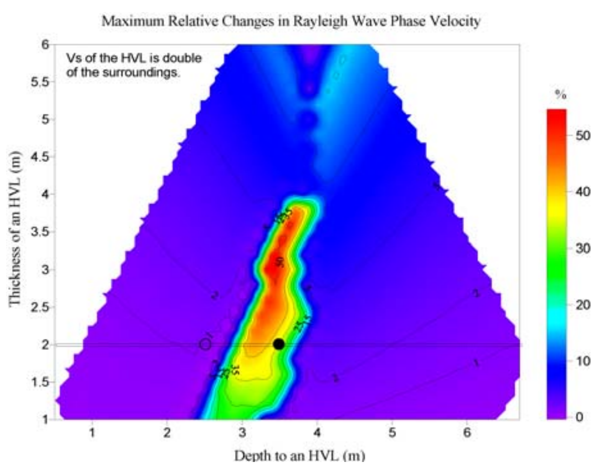

Figure 2. The maximum relative change with a 0.2 m change in depth to the center of an HVL (Vs = 400 m/s).

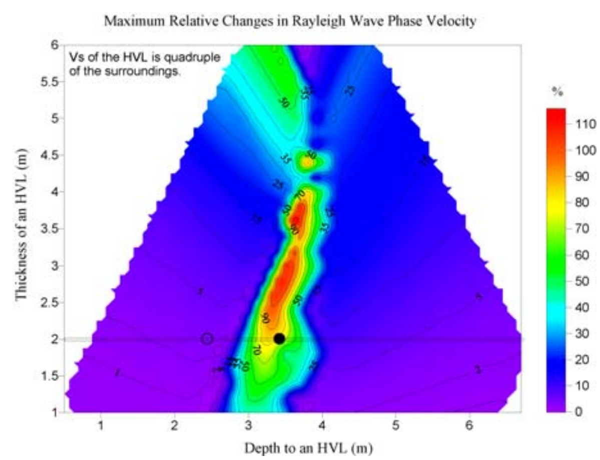

Figure 3. The maximum relative change with a 0.2 m change in a depth to the center of an HVL (Vs = 800 m/s).

the contrast between S-wave velocities of an anomalous layer and surrounding layers, the higher sensitivity of Rayleigh-wave data due to depth of the anomalous layer. Therefore, sensitivity of Rayleigh-wave data due to depth of an HVL is mainly dependent on the Vs contrast. We also used a model in Jin et al. (2007) to verify this point. When Vs of the HVL is reduced from 1500 m/s to 800 m/s, the maximum relative change reduces from 33% to 21%.

The sensitivity is also related to the depth of an HVL (Figure 4). For the 2-m thick HVL, the sensitivity reduces dramatically from 35% (80%) to 1% (2.5%) when depth changes from 3.4 m (a solid dot) to 2.5 m (a open dot) when its velocity is double (quadruple) of the surroundings. We observed the same with the model in Jin et al. (2007). The maximum relative change of 33% was observed at depth of 2.75 m with Jin's DP model while the maximum relative change was reduced to less than 1% at depth of 1 m.

Modeling results of an LVL

We also calculated the maximum relative change due to an

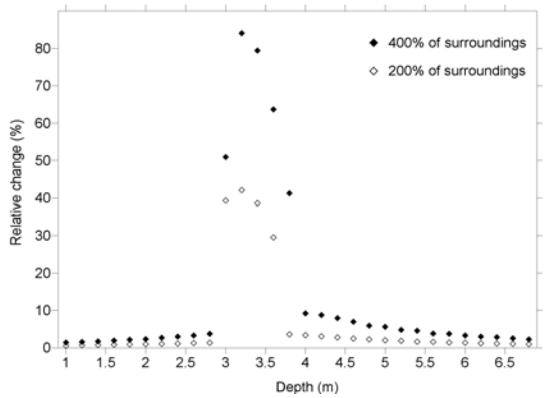

Figure 4. The maximum relative change with a 0.2 m change in depth to the center of the 2-m thick HVL. Note that the maximum change occurs around the center of top layers and is directly proportional to the Vs contrast.

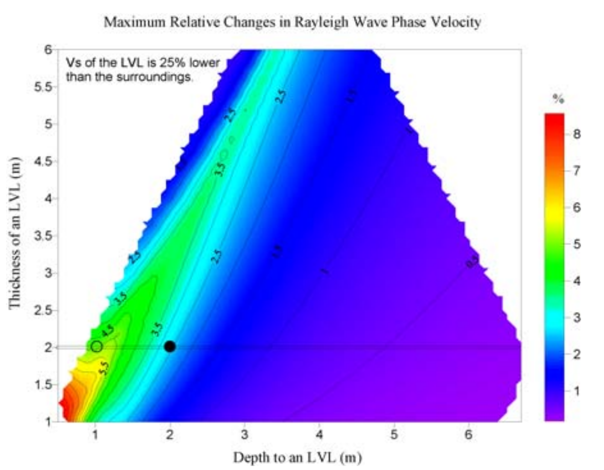

Figure 5. The maximum relative change with a 0.2 m change in depth to the center of an LVL (Vs = 150 m/s).

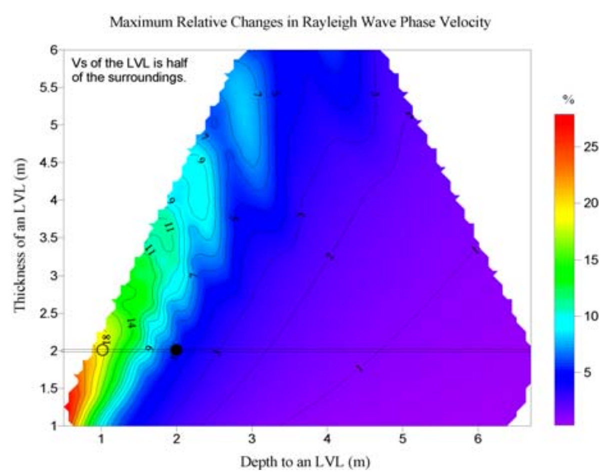

Figure 6. The maximum relative change with a 0.2 m change in depth to the center of an LVL (Vs = 100 m/s).

LVL in a homogeneous half-space (Figure 5). The setting is the same as the HVL case except for Vs = 150 m/s, Vp = Vs/0.2 and ρ = 1.2 g/cm^3 for the LVL. It is interesting to note that Rayleigh-wave data are most sensitive to an LVL at a depth near to the surface. When Vs of the LVL is reduced to 100 m/s (Vp = Vs/0.15 and ρ = 1.2 g/cm^3), the maximum relative change is more than doubled (Figure 6). The sensitivity increases from 2.5% (6%) to 5% (14%) when the depth changes from 2 m (a solid dot) to 1 m (a open dot) for Vs = 150 m/s (100 m/s). As in the HVL case, sensitivity is almost doubled at each depth for the 2-m thick LVL (Figure 7).

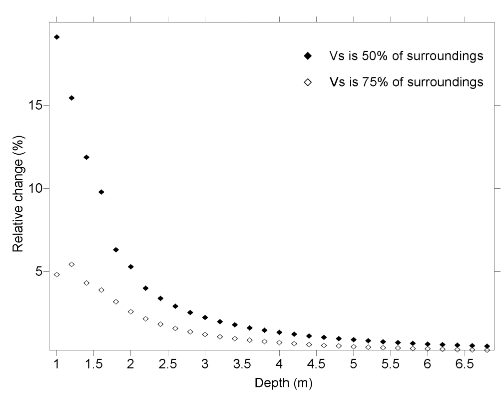

Figure 7. The maximum relative change in Rayleigh wave phase velocity due to a 0.2 m change in depth to the center of the 2-m thick LVL. Note that the maximum change occurs near the surface and is directly proportional to the Vs contrast.

Conclusions

In analysis of sensitivity of Rayleigh-wave data due to depth of layers, the Vs contrast is a dominant factor. Based on our modeling results for both HVL and LVL cases, we observed that the larger the Vs contrast is, the higher the sensitivity. The main difference between modeling results of HVL and LVL cases is that the most sensitive depth for an HVL is at around center of the depth to the half-space and but for an LVL it is near the ground surface. Implication of these observations is that it is much easier to define the depth of an HVL that is located at around the center of top layers and the depth of an LVL that is located near the ground surface.

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2007 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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