A LOCAL DAMPING SCHEME FOR NONREFLECTING BOUNDARY CONDITIONS IN SURFACE WAVE MODELING

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Summary

An efficient and robust artificial boundary condition is important for successful near-surface seismic modeling. Especially in near-surface seismic modeling related to surface wave application such as in the MASW (Multichannel Analysis of Surface Waves) method (Park et al., 1999), the significance becomes critical because correct amplitude and phase characteristics of surface waves have to be preserved without contamination by strong boundary reflections.

We apply a local damping scheme to the artificial boundary in finite-element modeling and test its possibility as a nonreflecting boundary condition. A local damping scheme dissipates seismic energy by inducing a damping force that is proportional in magnitude and opposite in direction to the total force induced on a node. This scheme shows compatible performance in suppressing spurious reflections from artificial boundaries compared with viscous damping and spatial filtering schemes. However, it is more efficient than these two schemes since it does not need extra grids for strips of damper that is unavoidable in the two schemes.

Introduction

An essential step for successful numerical modeling of seismic wavefields is to eliminate or suppress as much as possible reflections from the artificial boundary that are inevitable in solving a problem defined in an unbounded domain within a computational domain of finite dimension. Especially, in near-surface surface wave modeling where preserving correct amplitude and phase characteristics of surface waves is important, an effective and robust nonreflecting boundary scheme is required to suppress surface waves having relatively large amplitudes and wavelengths at the boundaries.

A variety of studies and reviews on nonreflecting boundary conditions have been reported. A common approach in geophysical application is to use one-way wave equation approximation (Engquist and Majda, 1977; Clayton and Engquist, 1977; Fuyuki and Matumoto, 1980; Emerman and Stephen, 1983). However, these methods highly depend on the type of wave and they break down for surface waves. In addition, instabilities associated with certain values of Poisson ratio and the angle of incidence have been reported by many researchers (Kelly and Marfurt, 1990). Some special techniques should be incorporated to handle surface waves properly (Joly, 1986; Liao et al., 1984).

A simple but effective alternative approach is a viscous damping or spatial filtering scheme (Lysmer and Kuhlemeyer, 1969; Cerjan et al., 1985, Kosloff and Kosloff, 1986; Sochaki et al., 1987; Sarmar et al., 1998). The viscous damping approach attenuates seismic wavefields by introducing a numerical viscous damper to the boundary or strip of damping zone surrounding the domain of interest to suppress reflection from artificial boundaries (Lysmer and Kuhlemeyer, 1969). The spatial filtering approach also artificially damps out wavefields within the damping region with artificial decay factors that have the largest values on internal boundary of sponge strip and taper gradually off towards the artificial boundary (Cerjan et al., 1985; Kosloff and Kosloff, 1986). These methods have been widely used due to their simplicity and effectiveness and are less dependent on the angle of incidence and the
type of waves. However, for the successful suppression of spurious reflections, both approaches require extra computational cost for sponge strips. Moreover, the damping effect must be smoothly applied over three to five wavelengths in thickness to avoid artificial scattering from sponge itself (Kelly and Marfurt, 1990). Such drawbacks often become serious burdens in a small-scale near surface seismic modeling for which the PC is usually used. To ensure sufficient suppression of surfaces waves having predominantly large amplitudes and wavelengths, the thickness of sponge strip should be thicker than usual, which inevitably degrades computational efficiency due to the reduction in the size of effective computational domain. Such extra computational cost becomes a huge burden in small-scale modeling of which the dimension of model usually does not exceed one hundred wavelengths. In our experience, the number of grids for the damping strip sometimes exceeds even a quarter of the total number of computational grids.

An alternative is to use the absorption parameters that are carefully selected to optimize the damping effect with a limited number of damping grids. To allow a quantitative analysis of damping parameters, Kosloff and Kosloff (1986) have extended their previous method (Cerjan et al., 1985) by introducing the concept of negative potential. Sochaki et al. (1987) have noted the difficulties of the method by Cerjan et al. (1985) in finding an optimum absorption coefficient due to its lack of underlying physical theory. They have presented a finite difference formulation fully based on the viscous damping approach and tested for several spatial damping functions. Sarma et al. (1998) have adopted a Rayleigh damping scheme in a finite element approach. To determine optimal interpolation coefficient for the damping matrix, they have empirically determined coefficients in terms of the frequency band of the source function instead of full eigenvalue analysis.

In this study, we adopt a local non-viscous damping—so-called local damping—as a boundary condition to explicit finite element modeling of elastic wave equation, and test the performance in suppressing spurious reflection from artificial boundaries.

Local Damping Scheme

Adaptive global damping has been known as an alternative that can overcome typical difficulties of standard velocity-proportional damping. For example, the standard velocity-proportional damping method requires a complete modal analysis to impose multiple proportionality constants on grids (Cundall, 1982). However, adaptive global damping continuously adjust viscosity constants in such a way that the ratio of the power absorbed by damping to the rate of change of kinetic energy is set to constant, i.e.,

$$R = \frac{\sum P_i}{\sum E_k},$$  \hspace{1cm} (1)

where $P$ is the damping power for a node, $\dot{E}_k$ is the rate of change of nodal kinetic energy, and $R$ is the damping ratio of the power (Cundall, 1982).

The local damping is similar to a local non-viscous version of adaptive global damping. Local damping dissipates energy by inducing a non-viscous damping force of which the magnitude is proportional to that of the unbalanced force on each node (Cundall, 1987). The direction of damping forces is such that energy is always dissipated, i.e.,

$$\mu_i = F - F_d$$

$$= F - \alpha|F|\text{sgn}(\dot{u})$$  \hspace{1cm} (2)

The absorption coefficient $\alpha$ is a constant ranging from 0 to 1 and describes the degree of energy dissipation. We note that equation (2) is similar to

$$m\ddot{u} + ku = -F_d \text{sgn}(\dot{u})$$  \hspace{1cm} (3)
that describes particle motion of a damped mass-spring system associated with Coulomb damping by dry friction. Therefore, *local damping* seems to be interpreted as energy dissipation by an internal friction mechanism. The magnitude of coefficient \( \alpha \) in equation (2) is dimensionless. Therefore, it is independent of properties or boundary conditions and can be set to vary from one grid point to another (Cundall, 1987).

**Numerical Examples**

**Rough Estimation of Absorption Coefficient**

Numerical implementation of *local damping* is easy and straightforward. However, selecting optimal coefficient \( \alpha \) is not so straightforward due to the lack of underlying physical theory for damping mechanism. As an empirical approach to find proper value of the coefficient, we compare results of the presented scheme for a given value of \( \alpha \) with those obtained from viscous damping similar to Shin (1995) and from the spatial filtering scheme by Cerjan et al. (1985).

Figure 1 illustrates the geometry of the homogeneous half-space model used in this study. The dimension of the model is 500 ft by 200 ft. The velocities of the medium are set to 3280 ft/s and 2296 ft/s for P- and S-wave velocities, respectively. The density of the medium is assumed to be 2.38 g/cm³. A vertical Ricker source is located at the center of the surface with maximum frequency of 100 Hz. Seismograms are gathered on the surface with receivers set at 5 ft intervals.

The *local damping* scheme is applied on left, right, and bottom boundaries of the model. For both spatial filtering and viscous damping schemes, sponge strip of 20 grids in thickness are set around left, right, and bottom boundaries, as depicted by the shaded region in the figure. Damping coefficients vary from 1.0 at the interior boundary of the strip to 0.98 at the boundary within the strip. For damper function, the exponential function proposed by Cerjan et al. (1985) is used.

Seismograms obtained from *local damping* with coefficient \( \alpha = 0.8 \) are presented in Figure 2, which shows the most similar results to those from both spatial filtering and viscous damping scheme shown in Figures 3 and 4. In Figure 5, traces at a location 100 ft away from the source are presented for detailed analysis. As noticed in this figure, results from all three approaches seem comparable to each other although the viscous damping approach gives slightly
improved performance. However, the viscoelastic approach changes the shape of the waveform as noticed around 380ms in enlarged display of figure (b). The effect of local damping shows very similar trend to Cerjan et al.’s (1985) approach. In Figure 5d, we can identify a bias effect toward negative amplitude of local damping. This may be attributed to abnormal damping forces on some grids. However, we note that local damping shows almost the same performance in suppressing artificial reflections without extra damping strips. In this experiment, the number of grids for damping strips takes about 24% of total number of computational grids despite the fact that this amount is not sufficient for better performance.

**Figure 3:** Seismogram obtained using spatial filtering scheme by Cerjan et al. (1985). (a) Horizontal displacement and (b) vertical displacement.

**Figure 4:** Seismogram obtained using viscous damping approach. (a) Horizontal displacement and (b) vertical displacement.
Effect of Absorption Coefficient

According to our experience, using an improperly large value for the absorption coefficient $\alpha$ may result in spurious reflections due to the boundary condition itself. In Figure 6, for example, seismograms obtained by applying the local damping scheme with the coefficient $\alpha=0.95$ are presented. Although overall performance has been improved, the reflections from the upper corners of the model become stronger than in the case of $\alpha=0.8$. This clearly shows that the coefficient $\alpha$ has to be optimized well to avoid spurious reflections from the boundary itself. We speculate that this might be associated with wavelength and amplitude of waves propagating along the surface.

Figure 6: Seismogram obtained using local damping boundary condition with $\alpha=0.95$. (a) Horizontal displacement and (b) vertical displacement.
Figure 7: Seismogram obtained using local damping scheme with spatial filtering scheme by Cerjan et al. (1985). (a) Horizontal displacement and (b) vertical displacement.

Figure 8: Comparison of (a) horizontal and (b) vertical displacements at a location 100 ft away from the source.

Figure 9: Seismogram obtained using local damping scheme with viscous damping. (a) Horizontal displacement and (b) vertical displacement.

Figure 10: Comparison of (a) horizontal and (b) vertical displacements at a location 100 ft away from the source.
Combination with Other Damping Schemes

A local damping scheme may be used in combination with other boundary conditions. Two combinations are tested: (a) local damping with conventional spatial filtering scheme by Cerjan et al. (1985) and (b) local damping with viscous damping. All parameters for each damping scheme are the same as those previously used. Results for the first combination (a) are shown in Figures 7 and 8, and those from the second combination (b) in Figures 9 and 10. As expected, results for both combinations show improved performance in suppressing spurious reflection over results obtained using each method separately. It is noted that the combination method suppressed spurious reflection up to nearly 100%. In addition, the bias effect of local damping shown in vertical displacement can be eliminated by applying other damping schemes together (Figures 8b and 10b). We expect better performance if we use optimized parameters for each damping scheme. However, there is a trade-off between the performance in suppression of artificial reflection and computational efficiency. For both combinations (a) and (b), computational efficiency will inevitably degrade due to extra cost for the damping strip for both viscous damping and a spatial filtering scheme.

Conclusions

We demonstrated that a local damping scheme could be an alternative nonreflecting boundary condition for wave equation modeling. The local damping scheme was shown to be effective and comparable to the other two damping schemes—viscous-damping and spatial filtering—without the computational cost of additional damping strips.

Since a local damping scheme is based on a damping approach, it shares in most of the advantages of a damping approach: simplicity and robustness. It can be used for both acoustic and elastic problems, and is less dependent on the type of waves, properties of the medium, and the angle of incidence. Moreover, it is more efficient than other damping approaches since it does not require extra computational grids for a damping strip.

The key to success in using a local damping scheme for a nonreflecting boundary condition is the selection of optimal absorption coefficients. The clue to solving this problem is better understanding of the underlying physical theory and numerical characteristics of the damping mechanism. Further research is required on these issues.

References


