Inversion of potential-field data using a hybrid-encoding genetic algorithm

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Summary

The genetic algorithm is of advantages to solve an inversion of complex non-linear geophysical equations. Its multi-point searching is able to find the globally optimal solution and avoid falling into a local extremum. The searching efficiency of the genetic algorithm is a key to successfully resolve a geophysical inversion problem in a huge model space with multi-parameters. Encoding mechanism impacts mostly in the searching stage of the genetic algorithm. It sometimes is difficult for a standard genetic algorithm (SGA) to make searching successfully, because the crossover and mutation do not receive most effectively searching in mechanisms with only the binary or decimal encoding. For the binary encoding mechanism the operation of the crossover may produce more new individuals. The decimal encoding mechanism, on the other hand, makes the operation of the mutation searching in a larger range. This paper discusses searching potentials between operators in the binary and decimal encoding and presents a hybrid-encoding genetic algorithm (HEGA) mechanism. The method is based on a hybrid encoding in genetic procedure. The mutation operation is executed with the decimal code and other operations with the binary code. The HEGA guarantees the mutation processing with a high probability. HEGA is beneficial to solving the inversion of complex non-linear geophysical equations. Synthetic and real-world examples demonstrated advantages of using HEGA in inversion of potential-field data.

Introduction

Many geophysical inverse problems are resolved using optimization. An objective function we are trying to optimize often is very complicated by multi-local extrema. Think of each trial solution as being a point in a ten-dimensional solution space, each coordinate is float-valued and can take on any value in its range. If we ponder a bit on whether or not the quantity we are trying to optimize is unimodal (has only one extremum) or not, we will be forced to the conclusion that in fact this function has no natural shape. The genetic algorithm, a "new" kind of global optimization algorithms, often referred to as GA, was presented in the late 1960s and early 1970s by Holland (1975) and his students. The mechanism that has "evolved" to carry out this search for the evolution problem consists of two parts, the genetic process and the natural selection. This mechanism has been manifestly successful. The application of the principles to the problem of geophysical inversion and data processing has become much more common (Berg, 1990, 1991; Farrell et al., 1996; Docherty, et al., 1997; Curtis and Snieder, 1997; Mallick, 1999; and Porsani and Ursin, 2000). In such systems pondering the speed of convergence to an optimum is inevitable since the randomicity of the mechanism. The improvements of the genetic algorithm for resolving special optimizing problems are commendable (Roth and Holliger, 1998; He and Fu, 1999).

Encoding techniques play an important role in the genetic evolution and influence searching and converging to an optimum. The dynamic parameter encoding technique gives more attempts to explore the "peak" of an optimized function (Schraudolph and Belew, 1992). Unique encoding system, however, does not make optimal approaches for all genetic operations. In this paper, we will discuss that the encoding mechanism with both binary and decimal codes, in which genetic operations are alternatively applied in the binary or decimal code, may create high potential to explore an optimal solution of a geophysical inverse problem.

Genetic searching

The operation of the genetic algorithm is based on a parameter encoding in the binary or decimal code. The binary encoding is used in the original genetic algorithm (Holland, 1975). The algorithm becomes more practical and easy to implement until the decimal encoding was introduced (Goldberg, 1989). Encoding mechanisms provide different situations for genetic searching (Zhang et al., 1997).

For the crossover operator the binary encoding supplies more opportunities of producing new strings than the decimal encoding. Suppose, an individual model consists of *m* parameters, which construct a string and the original colony with *n* individuals that are different form each other. Using x_t^i to denote the *i*th individual of the *t*th generation, we have

$$x_0^i \neq x_0^j, \quad i \neq j, \quad i, j \in (1,2,...,n) .$$

In the decimal encoding mechanism, total opportunities to produce new models (individuals) through crossover are given by

$$T_D = 2(m-1)C_n^2 = (m-1)(n-1) .$$

Changing encoding mechanism into binary encoding, each parameter can be expressed in the binary code with code-length of *L*. The amount of crossover-points increases from (*m*-1) to *m*(*L*-1). Therefore, total opportunities to produce new models through crossover in binary encoding mechanism are larger than or equal to those in decimal encoding mechanism, i.e., $T_B \geqslant T_D$.

The mutation operation is the secondary part for the genetic evolution, but it is necessary for generating new gene. The mutation operator in the binary code can only create limited opportunities to produce new models because of the disfigurement of the encoding mechanism. If a parameter data in the binary code with code-length of L goes through mutation (single-point mutation), the possibility of changing itself into the available numerical value within its range only is $L/(2^L\text{-}1)$. Thus, we cannot expect to access to every numerical value within the range of the parameter. For example, if the parameter with value of $(1011)_2=(11)_{10}$ gets through mutation, we probably get one of four "new" values $(1010)_2=(10)_{10}$, $(1001)_2=(9)_{10}$, $(1111)_2=(15)_{10}$, and $(0011)_2=(3)_{10}$. The rest of existent numerical values in its range, which the code-string is able to express, are immutable. The unavailable range may be called as "blind-zone".

Apparently, a parameter data can be changed itself into any numerical value within its rang through mutation in the decimal code based on the regulation of the decimal mutation operation (Goldberg, 1989).

Mechanism of hybrid-encoding genetic algorithm

The encoding mechanism of genetic algorithm influences the procedure of the evolution of individuals and colony since genetic operations are restricted in inadequate code systems. The binary encoding is an advantage for crossover searching and the decimal encoding makes the mutation more efficient in finding solutions. The analysis in the previous section gives a suggestion that introduces both of binary and decimal encoding systems to the procedure of evolution. The algorithm bases two encoding mechanism, i.e., a hybrid-encoding genetic algorithm (HEGA). The principle of the HEGA is that the mutation operation is executed in the decimal code and other operations in the binary code. In this procedure, more chances for the production of new models should be created.

The general operation stages of a hybrid-encoding genetic algorithm can be described as followings: encoding all parameters of models in the binary code, constructing model-space, generating original colony, operating selection and crossover under the probabilities desired, transforming parameter-codes of models selected to go through mutation from binary to decimal, operating mutation in the decimal code, transforming backward to binary, then getting in following evolution cycle.

Synthetic models

For the inversion problem of 2D gravity data, the shape of the object in the x-z plane can be described by polygons. The gravity anomaly is computed by giving coordinates on vertices of polygon (Talwani et al., 1959). The expression is written in

$$\Delta g(x,z) = 2G\sigma \sum_{k=1}^{n} \frac{\xi_k \zeta_{k+1} - \xi_{k+1}\zeta_k}{(\xi_{k+1}-\xi_k)^2 + (\zeta_{k+1}-\zeta_k)^2} [\frac{1}{2}(\zeta_{k+1}-\zeta_k)\ln\frac{\xi_{k+1}^2+\zeta_{k+1}^2}{\xi_k^2+\zeta_k^2} + (\xi_{k+1}-\xi_k)\mathbf{tg^{-1}}\frac{\xi_{k+1}}{\zeta_{k+1}} - \mathbf{tg^{-1}}\frac{\xi_k}{\zeta_k})]$$

where $\xi_k = x_k\text{-}x$ and $\zeta_k = z_k\text{-}z$, G is the gravitational constant, σ denotes the density contrast of the object, and x_k , z_k are coordinates of vertex k of the object section, respectively (Figure 1). When shifting to the last vertex in computation according to the equation, (x_{n+1}, z_{n+1}) should be replaced by (x_1, z_1). The equation shows that this is a complicated non-linear function for inversion although it has already used to resolve various forward modeling and inversion.

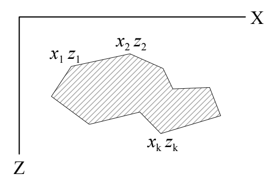

Figure 1. The object section of a horizontally polygonal column. (x_1, z_1), (x_2, z_2), …, (x_k, z_k) are coordinates of its vertices.

Three synthetic models with constant density contrast 1.0g/cm^3 are computed by the HEGA to illustrate the approach. Model-1 that is a hexagonal column with a general shape is inverted by using the SGA and the HEGA to compare their convergent speeds in evolved procedures. The same evolving strategy was used in both algorithms, which means keeping the probabilities of operations of crossover, mutation, and selection to the next generation, the size of colony and the rule of ending evolution are exactly the same. An inverted model is an average model from multi-trials. In our test, twenty trials are chosen for each method. We plot the average of square-root errors between parameters of model-1 and inversion results at each generation throughout the whole evolution (Figure 2). The average error is reduced steadily after a hundred generations by the HEGA. The convergence of the HEGA is more quickly than that of the SGA in this case. The outcome indicates the mutation in the decimal code plays an effective role in searching new parameters of the model.

In the practical world, we may not be able to know more details about our targets except density before the inversion. Thus, that real parameters of synthetic models or real models must be excluded from the initial colony will become a regulation in our trials. In other words, the initial colony does not include real solutions even the parameters of real solutions. It is predictable that convergent courses perhaps are not coincident under more than one trial. It is acceptable because the processing is random in many phases. With the same reason as the existence of the ambiguity of geophysical inversion, the final results fail to reach targets but can be somewhere close to targets. In this situation, an approximate but acceptable technique is to average the inverted model from desired trials, even though the best solution may be one of them.

Model-2 with higher density than surroundings possesses a lentoid shape like ore deposit shown in Figure 3. Its 20 parameters describing model need to be inverted. This optimization problem will be fixed in the twenty-dimensional space. Table 1 lists the average results of inversion by the HEGA under 20 trials. The inverted object is the closest to the model

(Figure 3). The mean square-root error is about 0.52 m and the average of relative errors is below 1 %. Model-3 is similar to Model-2, with higher density and a lentoid shape in the x-z plane, but its bottom boundary forms a convexity that makes the object shape complex. Choosing this model can inspect the resolution of the approach the HEGA for a complex model, especially the vertical resolution of the model. The result of inversion shown in Figure 4 is acceptable. The bottom boundary is definitely described in an acceptable accuracy.

A real-world example

High-resolution microgravity data were acquired in hope to detect the disused utility pipes and some unknown objects underground in a jobsite, Hubei, China. The surface of site is pretty flat (a relative elevation change is less than 0.5 m) and the bedrock lies about 15 m deep under the clay layer around the site. The microgravity data is obtained using LCR gravimeter (D-model) along a line cross the trend of the suspect sewer pipe and drainpipes. The Bouguer anomalies data after required corrections are shown in Figure 5. According to the shapes of gravity anomalies, we design three quadrangular horizontal columns to fit three local anomalies. The inverted models are obtained using the HEGA under the conditions: the depth of the sewer is within 0~8m, the depth of the drainpipe is within 0~3m, the depth of a hard object is within 0~2m, and the density contrasts is -2.1, 0.5, and -2.1g/cm^3 for these three targets, respectively. An adequately evolving strategy is necessary to get a truthful solution. The inversion presents three interpreted quadrangular objects that indicate two presumed objects and an unknown object. Objects indicated by inverted models are proved by later excavating. The hard object is a disused building base made of concrete.

Conclusions

Using a single encoding system to all genetic operations will limit their searching power in the application to resolve geophysical inversion problems. Introducing the hybrid-encoding mechanism into genetic algorithm is a deserved attempt. The synthetic and real-world examples demonstrated advantages of the HEGA in the mutation processing. It is important to point out that desired results should come from multi-trial by applying the genetic algorithm. More attempts can provide a better chance to obtain a truthful solution by averaging results from completed attempts. Our research in the aspect may provide a direction to explore evolution algorithms.

References

Berg, E., 1990, Simple convergent genetic algorithm for inversion of multiparameter data: 60th Ann. Internat. Mtg., Soc. of Expl. Geophys., 1126-1128.

Berg, E., 1991, Convergent genetic algorithm for inversion: 61st Ann. Internat. Mtg, Soc. of Expl. Geophys., 948-950.

Curtis, A., and Snieder, R., 1997, Reconditioning inverse problems using the genetic algorithm and revised parameterization: Geophysics, 62, 1524-1532.

Docherty, P., Silva, R., Singh, S., Song, Z.M., and Wood, M., 1997, Migration velocity analysis using a genetic algorithm: Geophys. Prosp., 45, 865-878.

Farrell, S.M., Jessell, M.W., and Barr, T.D., 1996, Inversion of geological and geophysical data sets using genetic algorithms: 66th Ann. Internat. Mtg., Soc. of Expl. Geophys., 1404-1406.

Goldberg, D.E., 1989, Genetic Algorithms in Search, Optimization and Machine Learning, Addison Wesley Publishing Company.

He, Q., and Fu, D., 1999, The improvement of genetic algorithm and its applications for the inversion of orthorhombic anisotropic media: 69th Ann. Internat. Mtg., Soc. of Expl. Geophys., 1791-1792.

Holland, J.H., 1975, Adaptation in natural and artificial systems: Univ. of Michigan Press.

Ji, Y., Singh, S., and Hornby, B., 2000, Sensitivity study using a genetic algorithm: inversion of amplitude variations with slowness: Geophys. Prosp., 48, 1053-1074.

Mallick, S., 1996, Prestack waveform inversion of the east Texas Woodbine gas sands using a genetic algorithm: 58th Mtg., Eur. Assn. Geosci. Eng., Session:C050.

Mallick, S., 1999, Some practical aspects of prestack waveform inversion using a genetic algorithm: An example from the east Texas Woodbine gas sand: Geophysics, 64, 326-336.

Porsani, M., and Ursin, B., 2000, Deconvolution and wavelet estimation by using a genetic algorithm: 70th Ann. Internat. Mtg., Soc. of Expl. Geophys., 2185-2188.

Roth, M. and Holliger, K., 1998, Joint inversion of Rayleigh and guided waves in high-resolution seismic data using a genetic algorithm: 68th Ann. Internat. Mtg., Soc. of Expl. Geophys., 1570-1573.

Schraudolph, N.N., Belew, R.K., 1992, Dynamic Parameter Encoding for Genetic Algorithm: Machine Learning, 9(9):9-21

Talwani, M.J., Worzel, L., and Landisman, M., 1959, Rapid gravity computations for two-dimensional bodies with application to the Mendocino submarine fracture zone: J. Geophys. Res., 64(1), 49-59.

Wilson, W.G., Laidlaw, W.G., and Vasudevan, K., 1994, Residual statics estimation using the genetic algorithm: Geophysics, 59, 766-774.

Zhang, X., Fang, H., and Dai, G., 1997, Study on encoding mechanism of genetic algorithm: Information and Control, 26, 134-139.

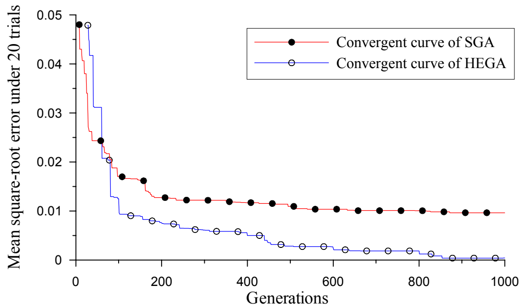

Figure 2. The convergent curves of inversion for model-1 by the SGA and the HEGA. The curve denoting the convergence in the HEGA keeps down after around a hundred generations. The error or fitness for the method of the SGA remains the same after about 200 generations and it seems hard to get convergence.

Table 1. Inversed coordinates of Model-2 and theoretical model are listed. The inverted model in table are the average of the results from 20 trials.

No.	Model		Inverted model	
	x (m)	z (m)	$\bar{x}$ (m)	$\bar{z}$ (m)
1	110	20	109.90	19.75
2	220	10	220.20	10.15
3	290	20	288.55	19.70
4	390	20	390.25	19.30
5	310	30	309.10	29.60
6	250	50	249.90	50.25
7	220	60	220.90	59.75
8	180	60	178.95	59.40
9	170	40	169.55	38.40
10	150	40	149.90	39.65

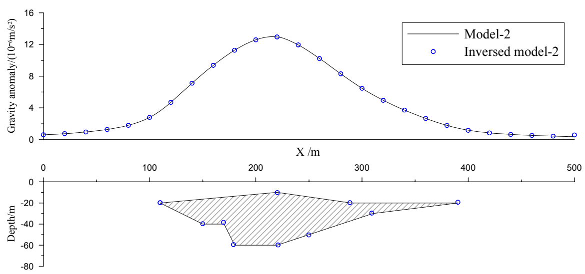

Figure 3. The inversion of model-2 using the HEGA. Computed anomaly from the inverted model exactly fit to the theoretical anomaly data from Model-2. The HEGA provides an accurate inverted model.

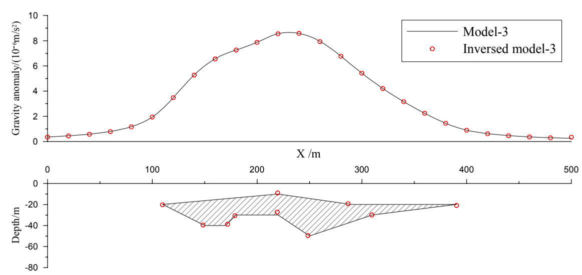

Figure 4. The inversion of model-3 using the HEGA. Computed anomaly from the inverted model fit the theoretical anomaly data from Model-3 with little error. The inverted model depicts Model-3 in detail, especially in the bottom boundary.

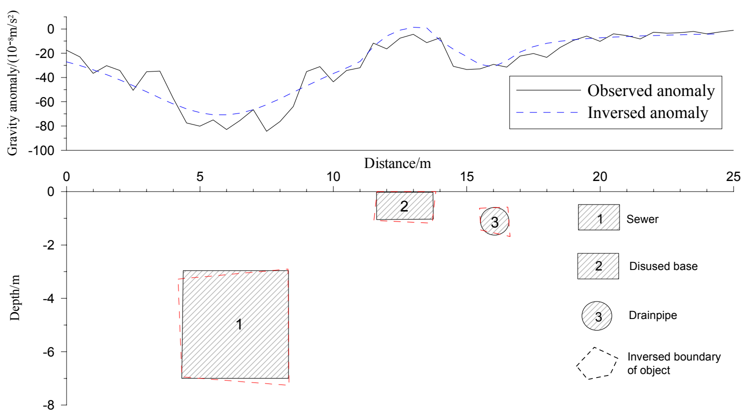

Figure 5. The inversion of gravity anomaly in a jobsite, Hubei, China. Since the maximum amplitude of anomaly just is about 50×10^{-8}m/s^{-2} (0.05 mGal), culture noise in observed data is obvious. Noise prevents theoretical anomaly data from perfectly fitting the observed data. Nevertheless, under the adequate restrictions and evolving strategies, the inverted models derived from the HEGA, successfully defined the targets.