Lamb waves observed during MASW surveys

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Abstract

Two fundamental modes of asymmetric and symmetric types of Lamb waves are observed from the near-surface seismic surveys by multichannel analysis of surface waves (MASW) method at two sites. One site consisted of loose soil charged with gas at shallow depth (at one and half meter). The other site was on top of hard, massive limestone possibly having a less stiff layer of shale or horizontal fracture at depth of approximately five meters. Lamb waves, originally misinterpreted as Rayleigh waves, were first identified from the prominent trend of both inverse (fundamental-mode asymmetric) and normal (fundamental-mode symmetric type) dispersion on the phase velocity image constructed from a multichannel analysis technique. Theoretical Lamb wave dispersion curves are then calculated and matched with the observed trend of dispersion on the image to calculate parameters such as thickness, and S- and P-wave velocities. These field examples indicate that Lamb waves can find useful applications in near-surface seismic surveying. Possible application may include detection of coal beds and plate voids as well as the cases illustrated here.

Introduction

Lamb waves are special type of elastic waves generated in a plate upon an impact on the surface when its upper and lower boundaries are free of traction (i.e., plate with traction-free boundary condition) (Lamb, 1917; Victorov, 1967) (Figure 1a). They form through a combination of multiple reflections and mode conversions between longitudinal (P) and shear (S) waves at the free surfaces. There are two types of propagation: symmetric (S) and asymmetric (A). Both propagate along the surface direction and are harmonic waves guided by two boundaries acting as a waveguide. In consequence, they are dispersive and multi-modal: different frequencies have different phase velocities, and each type has both fundamental and higher modes.

Although the original analysis of Lamb-wave dispersion (Kuttruff, 1991; Lamb, 1917) was based on the analysis of a plate with perfectly traction-free boundaries on both sides, later analyses and applications (Martincek, 1994) showed that Lamb waves can be generated even when one of the boundaries is not perfectly traction free. For example, even if one side of a plate is in contact with (or bounded by) another plate (or half space) of lesser stiffness (i.e., smaller Vs), Lamb waves can be still generated with only slightly different properties in dispersion and attenuation. In a pavement setting in which the uppermost layer of asphalt is bounded by an underlying base layer of granular material, the dispersion characteristics are virtually identical to the pure Lamb waves for those wavelengths as long as seven times the asphalt thickness (Martincek 1994).

In consequence, Lamb waves can be generated during the near-surface seismic surveys whenever there is a layer at depth whose shear-wave velocity (Vs) is significantly smaller than that of overlying layer(s). With this condition it should be noted that the thickness is not as important as the degree of the smallness of Vs and its two-dimensional (i.e., plate) characteristics. For example, if there is a layer at depth filled with gas, Lamb waves can be generated even if its thickness is small in comparison to its depth. Another example is the existence of a horizontal fracture within a consolidated rock where the stiffness virtually vanishes.

Because of the dispersive and multi-modal nature, the Lamb waves can be easily misidentified as Rayleigh-type surface waves during a near-surface seismic survey by, for example, multichannel analysis of surface wave (MASW) method (Park et al., 1999). Then, subsequent inversion process that assumes the measured curve as that of the Rayleigh waves will obviously give erroneous results. The purpose of this manuscript is twofold: first, to arouse an attention to this type of waves to minimize risk of misinterpreting them as surface waves, and, second, to introduce possible applications with Lamb waves in near-surface seismic investigation.

The application of ultrasonic Lamb waves has been extremely popular in material engineering during the last several decades (see, e.g., Kuttruff, 1991). Applications in this area include disbonding (Segal and Rose, 1980), and coating (Lee and Cheng, 2001) characterization of materials. Lamb-wave application in seismic exploration, however, has been rare. Some investigators used ultrasonic Lamb waves on thin rock plates to study certain rock properties such as porosity, permeability, and bulk velocities (Xue and Adler, 1990). Xu (1999) used Lamb waves of several kilohertz to study the effect of a vertical fluid-filled fracture on borehole logging. Yi et al. (1998) performed a numerical modeling of Lamb waves to investigate the propagation characteristics of elastic waves along the horizontal fracture planes. Its application to seismic pavement characterization has begun recently using a simulated multi-
Lamb waves in near-surface seismic surveying

channel approach (Ryden et al., 2002a; 2002b; Park et al., 2002).

Dispersion Properties of Lamb Waves

Dispersion characteristics of Lamb waves are determined by three parameters (Graff, 1975): thickness ($h$) and S- and P-wave velocities ($V_s$ and $V_p$) of the plate. At the highest frequency, phase velocities of all higher modes approach the shear-wave velocity ($V_s$) of the plate, whereas the fundamental modes ($A_0$ and $S_0$) approach the Rayleigh velocity ($V_r$). In consequence, from analysis of the Lamb-wave dispersion, one can deduce thickness ($h$) and S- and P-wave velocities ($V_s$ and $V_p$) of a plate.

![Diagram](attachment:image1.png)

Figure 1. (a) Schematic illustration of Lamb wave motion for symmetric (S) and asymmetric (A) types (from Vitorov, 1967). (b) First three modes of each type are calculated for a plate model shown inside the figure. This plate model is an identical one used for the modeling of field data shown in Figure 2a.

Figure 1a illustrates the wave motion of each type of Lamb waves within a plate (Kuttruff, 1991). Wave motion of each type is symmetric (S) and asymmetric (A) with respect to the center of the plate. S and A types have horizontal and vertical components, respectively, as major directions of motion. The fundamental mode of the symmetric type ($S_0$) approaches the velocity ($C_3$) of longitudinal wave in a two-dimensional medium at its lowest frequency (Martincek, 1994). $C_3$ changes with Poisson’s ratio ($\sigma$), and approach $V_p$, the bulk P-wave velocity, at its lowest frequency. However, $C_3$ is always smaller than $V_p$, and can be as small as 0.4$V_p$ when $\sigma$ is 0.48 (Martincek, 1994). Both types of all modes have phase velocities decreasing with frequency (normal dispersion) except for the fundamental mode of the asymmetric type ($A_0$) (Figure 1b). Phase velocity of $A_0$ always increases (from zero, in theory) with frequency (inverse dispersion), approaching Rayleigh-wave velocity ($V_r$). In practice, however, when the plate is bounded by underlying layer(s) of lower $V_s$, $A_0$ curve ends at a certain low frequency where its wavelength becomes significantly longer than plate thickness. Then, branching of dispersion curve takes place whose characteristics are determined by $V_s$ and $V_p$ of the underlying layer(s) (Vidale, 1964; Ryden et al., 2002a; 2002b). As frequency further decreases and wavelength becomes such a long that existence of the plate influences little on the wave phenomena, normal Rayleigh dispersion takes over as a governing relation.

Figure 1b shows dispersion curves of both symmetric and asymmetric types of Lamb waves for the plate model specified in the inset. The first three modes of both types are displayed in the figure. Relative amplitudes between different types and modes change with the plate parameters [i.e., thickness ($h$), $V_s$ and $V_p$], distance from source, and frequency (Graff, 1975). In general, the fundamental mode of the asymmetric type ($A_0$) dominates for a wide range of frequencies, whereas the symmetric type ($S_0$) usually gets its noticeable energy at a relatively narrow frequency range. In their investigation with pavement cases, Ryden et al. (2002b) showed that $S_0$ usually takes the most significant energy near the frequency range where $A_0$ approaches its asymptotic value (the Rayleigh wave velocity). They also showed that at most $A_0$ and $S_0$ are observed with prominent energy in most cases.

All these theoretical properties and field observations made on the Lamb waves indicate that whenever an inverse dispersion is identified without any other obvious reason, the Lamb wave nature should be suspected.

Lamb Waves Observed During Near-Surface Seismic Surveys

Two sets of field data are used to illustrate the occurrence of Lamb waves during MASW surveys. One data set (Figure 2a) was obtained during a surface-wave survey (MASW) over a soil site in Netherlands where the near-surface (< 5 m) seismic velocities are fairly low (approximately, $V_s=100$ m/sec and $V_p=500$ m/sec). This area, however, was found to have a shallow (1-5 m) gas-charged layer. The dispersion curve image (Figure 2b) of this data obtained from the wavefield-transformation method by Park et al. (1998) shows two dispersive seismic events. One event ($A_0$) shows the inverse dispersion, whereas the other
Lamb waves in near-surface seismic surveying

(S0) shows the normal dispersion. Therefore, the nature of seismic waves was later identified as Lamb waves, and dispersion curves were calculated for A0 and S0 with several different models of the uppermost soil layer. The pair of curves that had the best match was overlapped on the dispersion-curve image. Preliminary estimation of Vs was made based on the asymptotic value of A0 mode in the dispersion image, and this way of estimation is accurate usually with less than ten percent error. Similar estimation of Vp was made based on the image of S0 mode. However, a more accurate estimation was made based on the identification of a coherent arrival pattern in the time-domain display (shot gather) of the seismic record after a series of band-pass and F-k filtering had been applied. Significant discrepancy of the dispersion pattern to that of the pure Lamb case occurs at frequencies around 20 Hz (the portion of image marked as A). It is speculated at this time that this can be a smeared part of the dispersion image sometimes created when a significant amount of seismic energy is concentrated into a very narrow bandwidth (Park et al., 1998). It is also speculated that this may represent an actual dispersion behavior influenced by the gas-charged layer and layers further below. Similar feature is noticeable at frequencies around 35 Hz. The discrepancy also occurs when the frequency (wavelength) becomes small (long) (marked as B in the figure) where the Rayleigh waves from the underlying material start to influence (Ryden et al., 2002a; 2002b).

Another data set (Figure 3a) was collected on a hard surface during a MASW survey at a landfill near Kansas City, Kansas. Purpose of the survey was to locate a possible subsurface fracture zone at depths shallower than fifty meters. Each geophone was vertically placed on top of hard limestone with a cone-shaped plate replacing the spike. There was virtually no weathered zone at the surface, and the site geology consisted of typical Kansas cyclothems of massive limestone, or a horizontal fracture within a consolidated limestone, or a horizontal fracture within a massive limestone. Noticeable discrepancies between the two kinds of curves occur at low (< 50 Hz) and high (about 175 Hz) frequencies. They are explained in the same way as in the previous case of field data.

Discussions

Detailed investigation of Lamb-wave phenomena in association with near-surface seismic settings is currently under way as collaborative research between the Kansas Geological Survey (KGS) and Lund University, Lund, Sweden. At this moment, an elaborate analysis of field data is not possible. The influence of underlying material on the general characteristics of Lamb wave dispersion has to be dealt with before we proceed with full analysis.

Dispersion characteristics of Lamb waves are sensitive to the plate parameters. This indicates its possible application to resolve the elastic parameters of Vs, Vp, and Poisson’s ratio (ν) accurately.

Considering that Lamb waves are generated whenever there is a near-surface setting that can be approximated as a two-dimensional plate overlying a lower velocity materials, its immediate application may be the detection of shallow coal beds. Its possible application to the detection of shallow two-dimensional voids is currently under investigation.

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References

Lamb waves in near-surface seismic surveying


Figure 2. (a) A shot gather acquired at a soil site charged with gas at shallow depth in Netherlands. Vertical source and receivers were used. (b) The dispersion curve image processed from (a) using a wavefield transformation method by Park et al. (1998). Two theoretical curves of Lamb waves calculated for the fundamental modes of asymmetric (A0) and symmetric (S0) types are overlaid. The plate model used for the calculation is shown in inset. Other marked features (A and B) are explained in the main text.

Figure 3. (a) A shot gather acquired on a hard limestone surface at a landfill near Kansas City, Kansas. Vertical source and receivers were used. (b) The dispersion curve image processed from (a). Two theoretical curves of Lamb waves calculated for the fundamental modes of asymmetric (A0) and symmetric (S0) types are overlaid. The plate model used for the calculation is shown in inset. Other marked feature is: B-Rayleigh-type surface waves generated from the lower layers.