Feasibility of determining *Q* of near-surface materials from Rayleigh waves

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Summary

High-frequency (5–35 Hz) Rayleigh waves phase velocities can be inverted to shear (S)-wave velocities for a layered earth model up to 30 meters below ground surface in many settings (Xia et al., 1999 and Park et al., 1999). Given S-wave velocity, compressional (P)-wave velocity, and Rayleigh wave phase velocities, it is feasible to solve for P-wave quality factor Q_P and S-wave quality factor Q_S in the layered earth model by inverting Rayleigh wave attenuation coefficients. Model results demonstrate the plausibility of inverting Q_S from Rayleigh wave attenuation coefficients. Contributions to the Rayleigh wave attenuation coefficients from Q_P cannot be ignored when Vs/Vp reaches 0.45, which is not uncommon in near-surface settings. It is possible to invert Q_P from Rayleigh wave attenuation coefficients in some unconsolidated materials, which is a concept that differs from the common perception that Rayleigh wave attenuation coefficients are always far less sensitive to Q_P than to Q_S (Mitchell, 1975).

Introduction

The most common measure of seismic wave attenuation is the dimensionless quality factor *Q* and its inverse (dissipation factor) Q^{-1}. As an intrinsic rock property, *Q* represents the ratio of stored to dissipated energy (Johnson and Toksöz, 1981). The quality factor as a function of depth is of fundamental interest in groundwater, engineering, and environmental studies, as well as in oil exploration and earthquake seismology. Motivation to understand the attenuative properties of the earth are based in the observation that seismic wave amplitudes reduce as waves propagate through an elastic medium. This reduction is generally frequency dependent and, more importantly, attenuation characteristics can reveal unique information about lithology, physical state, and degree of rock saturation (Toksöz and Johnson, 1981).

Estimates of the shear (S)-wave velocity (Vs) from ground roll, a particular type of Rayleigh wave, and how to apply in the real world has been extensively investigated (Xia et al., 1998, 1999, 2000a, and 2000b; Park et al., 1999; and Miller et al., 1999). Based on documented experiences, when the fundamental-mode phase velocities are calculated with a high degree of accuracy (e.g., Xia et al., 2000a) reliable S-wave velocities (±15%) can be estimated. Incorporating higher mode data into the surface wave analysis increases the resolution (or accuracy) of the inverted S-wave velocities (Xia et al., 2000b). After successfully determining a near-surface V_S profile from Rayleigh waves, the feasibility of calculating *Q* from Rayleigh wave attenuation coefficients can be analyzed.

Laboratory experiments show that *Q* may be independent of frequency over a broad bandwidth (10^{-2}–10^{7} Hz), especially for some dry rocks. Q^{-1} in liquids is proportional to frequency, however. Q^{-1} may contain a frequency dependent component that is negligible at seismic frequency, even in unconsolidated marine sediments (Johnson et al., 1981). Although some authors suggest that near-surface *Q* may be frequency-dependent (Jeng et al., 1999), we will assume that *Q* is independent of frequency, allowing determination of *Q* as a function of depth based on amplitude attenuation of Rayleigh wave data. In this paper we will examine the relationship between Rayleigh wave attenuation coefficients and compressional (P)-wave and S-wave quality factors (Q_P and Q_S) through forward modeling. The modeling will be used to develop a quantitative description of the contributions to Rayleigh wave attenuation coefficients from Q_P and Q_S.

Basic Equations

To determine *Q* as a function of depth in near-surface materials (up 30 meters), utilization of high-frequency Rayleigh waves (5–35 Hz) is essential. The relationship between Rayleigh wave attenuation coefficients and the quality factors for compressional and shear waves were given by Anderson et al. (1965) as:

$$\alpha_R(f) = \frac{\pi f}{C_R^2(f)}\left[\sum_{i=1}^{n} P_i(f) Q_{Pi}^{-1} + \sum_{i=1}^{n} S_i(f) Q_{Si}^{-1}\right]. \tag{1}$$

where

$$P_i(f) = V_{Pi}\frac{\partial C_R(f)}{\partial V_{Pi}}, \tag{2}$$

$$S_i(f) = V_{si}\frac{\partial C_R(f)}{\partial V_{Si}}, \qquad (3)$$

$\alpha_R(f)$ is Rayleigh wave attenuation coefficients in 1/length, and f is frequency in Hz. Q_{Pi} and Q_{Si} are the quality factors for compressional and shear waves of the *i*th layer, respectively. V_{Pi} and V_{Si} are the P-wave velocity and S-wave velocity of the *i*th layer, respectively. C_R is Rayleigh wave phase velocity. n is the number of layers of a layered earth model.

Attenuation coefficients can be determined using the relationship between amplitudes (A) and attenuation coefficients: $A \propto \exp[-\alpha_R(f)x]$, where x is the distance between a source and a receiver (Kudo and Shima, 1970). Because Equation (1) is a linear system, the same method used in Xia et al. (1999) can be employed directly to solve Q_P and/or Q_S from Rayleigh wave attenuation coefficients. In many cases, only a single iteration is necessary for convergence.

Equation (1) manifests the linear relationship between Rayleigh wave attenuation coefficients and the dissipation factors for compressional and shear waves (Q_P^{-1} and Q_S^{-1}). Theoretically, after determining S-wave velocities by inverting Rayleigh wave phase velocities (Xia et al., 1999 and Park et al., 1999) and P-wave velocities by other seismic methods, such as reflection (Hunter et al, 1984; Steeples and Miller, 1990) refraction methods (Palmer, 1980), and/or tomography (Ivanov et al., 2000), the dissipation factors (Q_P^{-1} and Q_S^{-1}) can be inverted directly for noise-free data using Equation (1). Practically, however, common thought suggests that surface wave attenuation is far less sensitive to Q_P than to Q_S (Mitchell, 1975), making it difficult to invert Q_P from Rayleigh wave attenuation coefficients.

In the following section, the sensitivity of Rayleigh wave attenuation coefficients with respect to the dissipation factors, Q_P^{-1} and Q_S^{-1} will be analyzed. Using forward modeling, it will be demonstrated that in some cases Q_P can be obtained from inverting Rayleigh wave attenuation coefficients, just like Q_S.

Modeling Results

Equations (2) and (3) represent the rate of change of Rayleigh wave attenuation coefficients $\alpha_R(f)$ to dissipation factors Q_P^{-1} and Q_S^{-1} of the *i*th layer, respectively. P_i is the product of the P-wave velocity of the *i*th layer and the partial derivative of Rayleigh wave phase velocities with respect to P-wave velocity of the *i*th layer. S_i is the product of the S-wave velocity of the *i*th layer and the partial derivative of Rayleigh wave phase velocities with respect to S-wave velocity of the *i*th layer. P_i and S_i totally control the sensitivity of Rayleigh wave attenuation coefficients to Q_P^{-1} and Q_S^{-1}.

The six-layer model (Xia et al., 1999) is employed to analyze contributions to Rayleigh wave attenuation coefficients from Q_P and Q_S (Figure 1). Letting V_S change from 25% of V_P to 50% of V_P, contributions of Q_P to Rayleigh wave attenuation coefficients increase with increasing Vs/Vp while Q_S contributions to Rayleigh wave attenuation coefficients decrease as Vs/Vp increases (Figure 2). Q_P contributions become significant for almost all frequencies when Vs/Vp approaches 0.45. For example, for the 30 Hz component, when Vs/Vp is 0.5, Q_P contributions dominate and reach more than 70% while Q_S contributions fall to less than 30%. Roughly speaking, when Vs is about one half Vp, overall contributions of Q_P to Rayleigh wave attenuation coefficients may reach more than 30%. This result suggests it may be possible to invert Q_P from Rayleigh wave attenuation coefficients when Vs is approximately one half of Vp.

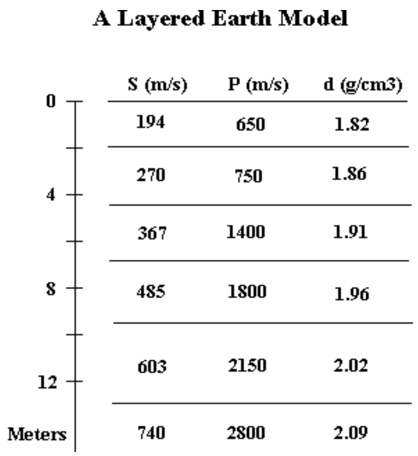

Figure 1. A layered earth model (Xia et al., 1999) is used to analyze the relationship shown in Equation (1).

Sensitivity of Rayleigh wave attenuation coefficients to Q_P and Q_S is evident in the model (Figure 1) when Vs/Vp is 0.5. For a dry sandstone, Q_P/Q_S is almost equal to one (Johnson, 1981), making Q_P and Q_S equal (5, 10, 12, 15, 20, and 25 from the top

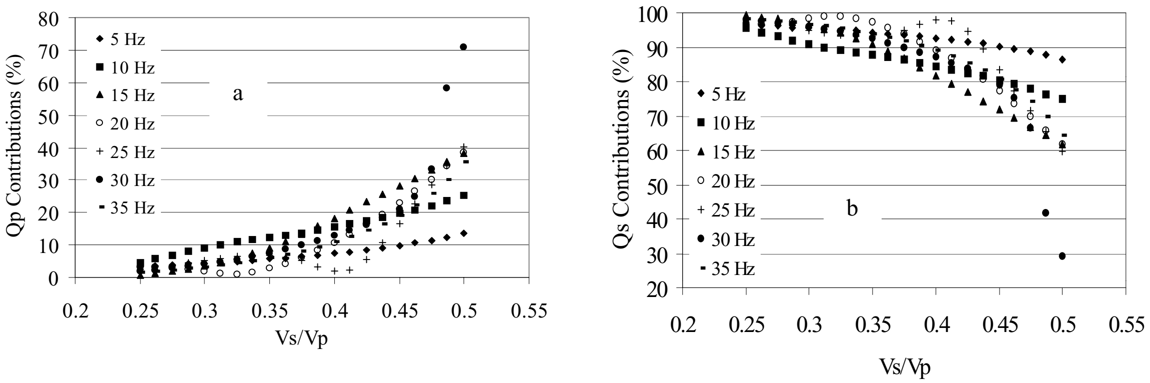

Figure 2. Q_P contributions to Raleigh wave attenuation coefficients (a) and Q_S contributions to Raleigh wave attenuation coefficients (b). Q_P contributions are dominant when Vs/Vp is about 0.5.

layer to the half space). With a 25% reduction in Q_P and/or Q_S (3.75, 7.5, 9.0, 11.25, 15.0, and 18.75 from the top layer to the half space) results in the relationship shown in Figure 3. With a 25% reduction in Q_P, the relative increases in Rayleigh wave attenuation coefficients are in the range 4–20%, averaging 12% from 5 to 35 Hz. For the same reductions in Q_S, the relative increases in Rayleigh wave attenuation coefficients are in range 9 to 23% with an average of 17% from 5 to 35 Hz. The overall relative increases in Rayleigh wave attenuation coefficients due to 25% reductions in both Q_P and Q_S are almost the same at 28% in relative change within the 5 to 35 Hz frequency range. For a water saturated sandstone, Q_P/Q_S may reach 2 (Johnson, 1981). In that case, the contributions to Rayleigh wave attenuation coefficients due to Q_P may surpass those due to Q_S.

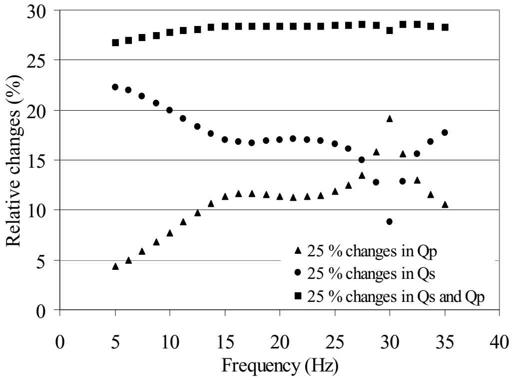

Figure 3. Sensitivity of Rayleigh wave attenuation coefficients to Q_P and Q_S.

Discussion and Future Study

Quality factor Q_P may be inverted when Vs/Vp being greater than 0.45, a situation which is common in oil industry and crust seismology studies, and which is also not uncommon in near-surface materials. Based on sensitivity analysis in the last section, errors in inverted quality factors can reach 1 to 1.5 times the errors in attenuation coefficients. Compared to the system that Xia et al. (1999) used to invert Vs from Rayleigh wave phase velocities, Equation (1) has less stability, making accurate calculation of Rayleigh wave attenuation coefficients critical. On the other hand, the system (Equation 1) we develop here is more stable than AVO analysis routinely used in the oil industry (e.g., Jin et al., 2000). The proposed technique is being tested with real world examples.

Acknowledgments

The authors thank Mary Brohammer for her assistance in manuscript preparation and submission.

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