HUM FILTER: POWER-LINE NOISE ELIMINATOR FOR SHALLOW SEISMIC DATA

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ABSTRACT

A unique filtering approach designed to eliminating power-line noise on shallow seismic data without affecting the frequency content of signal provides a powerful harmonic noise suppression tool for data acquired with modern large dynamic range recording systems. Amplitudes and phases of sinusoids (functions of power-line noise) present before the first arrivals can be estimated using the Levenberg-Marquardt (L-M) method. Initial amplitudes of sinusoids are determined in the frequency domain using fast Fourier transform (FFT) methods while initial phases are obtained by time domain correlation. Well-defined initial values guarantee convergence of the L-M method. Modeling results suggest the relative error of initial estimates are less than 50 percent. Calculation efficiency is achieved by simplifying the L-M solution using the singular value decomposition (SVD) technique. The approach can handle cases where power-line noise with frequencies of 60 Hz and/or its multiples exist simultaneously. Once determined, the amplitudes and phases of sinusoids can be directly subtracted from the raw data. Recorded frequencies of high-resolution shallow seismic surveys generally range from 30 to 300 Hz. Power-line noise (60 Hz) and its multiples (120 Hz, 180 Hz, and 240 Hz, etc.) are within the optimum frequency range. This filtering technique only removes harmonic noise and does not alter the spectra of signal. Real data examples demonstrate the efficiency and accuracy of this method when implemented on a normal shallow seismic data processing flow.

INTRODUCTION

Single-frequency periodic interference such as power-line noise, routinely plagues high-resolution shallow seismic surveys. The conventional tool used to eliminate power-line noise is a notch filter applied during data acquisition or data processing. A major drawback of this practice is that all frequency components around 60 Hz are muted regardless if they are signal or noise after applying a notch filter. Recorded frequencies of high-resolution shallow seismic surveys generally range from 30 to 300 Hz. Sixty hertz power-line noise and its multiples (120 Hz, 180 Hz, and 240 Hz, etc.) are within the optimum frequency range. It is not uncommon to see multiples of 60-Hz interference in high-resolution shallow seismic data (e.g., Miller et al., 1994). Application of a notch filter generally results in a saw tooth shaped spectrum with drop-outs around the notch frequency.

Black (1997) suggested that 60-Hz noise may be eliminated by *f-k* filtering techniques. If the noise is not in phase across several traces, it may be possible to save 60-Hz reflection energy near the *f* axis in *f-k* space, and suppress the majority of the 60-Hz noise at higher *k* numbers away from the *f* axis in *f-k* space. If the power-line noise is in-phase, *f-k* filtering techniques will suppress the 60-Hz component of signal as well as noise. Real data examples demonstrate that power-line noise can be in-phase for several traces and out-of-phase for other traces on one shot gather (see figure 2a). This inconsistently in trace-to-trace phase will adversely affect a *f-k* filter ability to remove power-line noise.

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There are several other techniques developed to eliminate power-line noise specifically vibroseis data. Crook (1966) and Wischmeyer (1966) suggested a pseudorandom code generated by multiplying a constant-frequency carrier signal by a pseudorandom sequence of binary elements. Cunningham (1977, 1979) presented a method to reduce bothersome single-frequency interference, such as 60-Hz power-line noise, by simply choosing a carrier frequency which coincides with the interference frequency. The amplitude of the single frequency of the psuedorandom code is nulled in the frequency domain because of the psuedorandom code (assumed to be random). The amplitude of the correlated results from the psuedorandom code and received signal at the carrier frequency will be zero. Obviously, the 60-Hz spectral component of actual signal is eliminated as well. This techniques only works for single-frequency interference.

All these techniques suffer from one drawback: signal is removed with noise. We have designed a hum filter based on the power-line noise present before first arrivals. Power-line noise can be thought of as a summation of sinusoid functions with different frequencies, amplitudes, and phases. The technique can easily handle 60-Hz noise and its multiples. Initial amplitudes of sinusoids are determined in the frequency domain by fast Fourier transform (FFT) methods and initial phases by a time-domain correlation. The Levenberg-Marquardt (L-M) method (Marquardt, 1965) is used to update amplitudes and phases of sinusoids (functions of power-line noise). Calculation efficiency is achieved by using the singular value decomposition (SVD) techniques (Golub and Reinsch, 1970), which simplifies the L-M solution. Power-line noise is eliminated by subtracting the modeled sinusoids in the time domain. This new approach only removes harmonic noise and does not alter the spectra of signal.

THE METHOD

The hum filtering method will be developed in three parts: power-line noise model, methods to estimate initial values of amplitudes and phases, and formulation an inversion algorithm by the L-M method and the SVD technique to modify initial values of amplitudes and phases.

1. Power-line Noise Model

Power-line noise is modeled from energy recorded before the first source generated energy arrivals. We define the power-line noise model as a summation of sinusoids

$$P(\vec{x},t) = \sum_{i=1}^{n} A_i \sin(2\pi f_i t + \Phi_i)\,, \tag{1}$$

where P is a power-line noise model, t is time, A_i and Φ_i are the amplitude and phase of power-line noise with a frequency of f_i, respectively, n is the number of sinusoid functions that are needed to model power-line noise, and the vector $\vec{x}$ represents the parameters of the power-line noise model ($\vec{x} = [A_1, \Phi_1, A_2, \Phi_2, \ldots, A_n, \Phi_n]^{\mathrm{T}}$). Determining n requires analysis of the spectral properties of the raw data. In our research, n has normally been less than four. The total number of parameters in the model is $2\times n$ (the parameters for each sinusoid are the amplitude and phase).

2. Initial Amplitudes and Phases

Due to the number of samples of each trace and the limitation of FFT techniques, the frequencies of a summation of sinusoids (Eq. 1) are usually not exactly sampled in the frequency domain. The initial amplitude estimates of each sinusoid are, therefore, based on the amplitude of the frequency closest to the sinusoid frequency. Our numerical experience show that relative errors of initial estimates determined by this simple method are less than a 50 percent.

Cross correlation between power-line noise present prior to the first arrivals and a sinusoid of a given frequency in the time domain is the approach used to estimate initial values of the phase for power-line noise at a given frequency (Eq. 1). The time lag with the maximum cross-correlation coefficient will be defined as the initial phase. Because the time lag of the cross correlation can be a fraction of the sample interval, the initial estimates of phases can be determined within the same accuracy range as that for amplitudes.

Cross correlation in the time domain became the preferred approach after attempts to determine the initial values of phases in the frequency domain proved unsuccessful. For the same reason that the frequencies of a summation of sinusoids (Eq. 1) are usually not exactly sampled in the frequency domain, initial values of phases cannot be determined in the frequency domain. Based on our experience, relative errors of initial estimates of phases when estimating phase in the frequency domain were as large as 200 percent. This problem is compounded by the fact that there is no way to determine the sign (positive or negative) of phase values when examining the discrete spectra calculated by FFT.

Initial values determined using FFT techniques for amplitudes and cross-correlation in the time domain for phase have a 50 percent relative accuracy. Although a 50 percent relative error is too high to formulate a power-line noise model (Eq. 1), our numerical experience suggest 50% is good enough to start a modification algorithm designed to update initial amplitudes and phases.

3. Modification Algorithm

There are $2\times n$ parameters that need modification to accurately define power-line noise model (Eq. 1). These parameters are A_i and Φ_i where i = 1, 2, ..., n. We use the vector $\vec{x}$ to represent these parameters $\vec{x} = \left[A_1, \Phi_1, A_2, \Phi_2, \ldots, A_n, \Phi_n\right]^T$. Similarly, noise present before the first arrivals can be represented as the vector $\underline{b}$ of length m, $\vec{b}$ = [$\underline{b}_1$, $\underline{b}_2$, $\underline{b}_3$, ..., $\underline{b}_m$]^T.

The objective function is defined as

$$\Psi = \left\|\mathbf{J}\,\Delta\vec{x} - \vec{b}\right\|_2^2 + \lambda\left\|\Delta\vec{x}\right\|_2^2, \tag{2}$$

where λ is a damping factor, $\vec{b} = \underline{b} - P(\vec{x}_0)$, $\vec{x}_0$ is an initial estimate of parameters of the power-line noise model, and $\mathbf{J}$ is the Jacobi matrix consisting of m rows and $2\times n$ columns. The elements of the Jacobi matrix are the first order partial derivatives of P with respect to parameters of the power-line noise model at each sampling time. Elements of the Jacobi matrix are sine and cosine functions and easily calculated.

Since the number of samples preceding the first arrivals is generally more than the number of parameters in the power-line noise model, Eq. 2 can be solved using optimization techniques. The damping least-squares solution to Eq. 2 can be determined from the normal equation $(\mathbf{A}^T\mathbf{A}+\lambda\mathbf{I})\Delta\vec{x} = \mathbf{A}^T\vec{b}$. After applying the SVD technique to the normal equation, we obtain the solution to Eq. 2

$$\Delta\vec{x} = \mathbf{V}(\Lambda^2+\lambda\mathbf{I})^{-1}\Lambda\mathbf{U}^T\vec{b}, \tag{3}$$

with $\mathbf{V} = [\vec{v}_1, \vec{v}_2, \vec{v}_3, ..., \vec{v}_{2n}]$ where $\vec{v}_i (i = 1, 2, ..., 2\times n)$ are eigenvectors of matrix $\mathbf{A}^T\mathbf{A}$ and $\Lambda = \mathrm{diag}\,(\sqrt{\lambda_1}, \sqrt{\lambda_2}, ..., \sqrt{\lambda_{2n}})$. This diagonal matrix has elements that are the square root of the eigenvalues of $\mathbf{A}^T\mathbf{A}$, $\mathbf{U} = [\vec{u}_1, \vec{u}_2, \vec{u}_3, ..., \vec{u}_{2n}]$, $\vec{u}_i = \mathbf{A}\vec{v}_i / \lambda_i$ $(i = 1, 2, ..., 2\times n)$, and $\mathbf{I}$ is the unit matrix. Evident is the dependence of Eq. 3 on the damping factor λ. Marquardt (1965) pointed out that the damping factor controls the direction of $\Delta\vec{x}$ and speed of convergence. Eq. 3 is an interpolation between the least-squares solution and the steepest descent solution. Several different values of λ are necessary to insure the damping factor is optimized. Directly inverting the matrix $(\mathbf{A}^T\mathbf{A}+\lambda\mathbf{I})$ is not an efficient way to search for the proper damping factor. The inverse matrix of $(\mathbf{A}^T\mathbf{A}+\lambda\mathbf{I})$ must be calculated every time the damping factor λ is changed. Because the inverse matrix in Eq. 3 is a diagonal matrix, inverting this matrix requires simple division. Therefore, only matrix multiplication is needed to determine $\Delta\vec{x}$ for each new damping factor.

After determining $\Delta\vec{x}$, the initial amplitude and phase estimates are modified by

$$\vec{x}^k = \vec{x}^{k-1} + \Delta\vec{x}. \tag{4}$$

Updating is terminated when the root-mean-square (RMS) error between model values (Eq. 1) and power-line noise is reduced to a pre-defined threshold or the number of iterations reaches a pre-defined limit.

4. Subtracting Power-line Noise Model from Data

Power-line noise can be calculated at any time after determining amplitudes and phases of the power-line noise (Eq. 1). Calculated noise can be subtracted from the data. Theoretically only power-line noise is attenuated while signal containing frequencies that are the same as that of the power-line noise is untouched.

Subtraction of modeled noise (Eq. 1) should be applied only to traces that contain power-line noise. In practice, this can be handled by checking the percentage reduction in the RMS error at the end of the modification procedures. Thirty percent reduction in the RMS error is required, in our experience, to allow adequate subtraction of modeled noise to be applied to data.

REAL WORLD EXAMPLES

High resolution seismic reflection data at Memphis Defense Depot, Memphis, Tennessee were acquired on 48-channel EG&G Geometrics 2401X seismograph (Miller, et al., 1994). Three Mark Products L-28E 40 Hz geophones wired in series were deployed per station. Geophone station space was 2.5 m. The source was the 8-gauge auger gun. Data were acquired using a 1/2 msec sampling interval which provided a 2000 Hz sampling frequency over a 500 msec record. A 60 Hz notch filter and a 250 Hz high-cut filter were applied to raw data during acquisition to reduce 60 Hz noise and its multiples.

A 60 Hz notch was applied during data acquisition. Therefore, only power-line noise with 180 Hz and 300 Hz needed to be attenuated. The first 10 msec of data (20 samples) were used to model power-line noise. A Pentium-155 PC required 1.6 seconds to process one shot gather (figure 1a). The results of the hum filter (figure 1b) show the effectiveness of this filtering technique. Using the first 50 msec of data (100 samples) to model the power-line noise produced a section almost identical to the section produced using only the first 10 msec. This suggests the filter is not particularly sensitive to the length of the time window.

Spectra of selected traces before (figure 1c) and after (figure 1d) the hum filter dramatically demonstrates the effectiveness of the filter. A notch in the frequency-amplitude plot around 60 Hz (Figure 1c) is the remnant of the notch filter applied during data acquisition. Signal as well as noise around 60 Hz were filtered out. After the hum filter, 180 Hz noise was reduced more than 40 times from its original amplitude and the 300 Hz noise was reduced to less than one quarter of its original amplitude.

Development of this filtering technique was prompted by high concentrations of power-line noise (60 Hz and its multiples) prominent across the entire spectra of data acquired along a 22-mile long high resolution seismic reflection survey near Zeandale, Kansas (figures 2a and 3a) (Feroci et al., 1997). Uncorrelated vibroseis data were acquired on a 96-channel Geometrics StrataView seismograph. A total of 81 channels were recorded using assymmetric split-spread for each shot. The receivers were three Mark Products L-28E 40 Hz geophones wired in series. Geophone station and source spacing was 16.5 m. The source was the IVI Minivib running a 10 sec linear up-sweep from 20 Hz to 200 Hz. The 1 msec sampling interval resulted in a 1000 Hz sampling frequency. A 12 second record provided a 1000 msec shot gather after cross-correlation. Notch filtering was not necessary during acquisition due to the degree of success expected from this hum filter removing the power-line noise.

Shot gathers from Zeandale have traces that appear saturated with 60 Hz and 180 Hz power-line noise (e.g., figure 2a). The spectrum of selected traces of this shot gather demonstrates the magnitude of the power-line noise problem (figure 2c). The same shot gather after hum filtering (figure 2b) and spectral calculations (figure 2d) demonstrate the noise suppression capabilities. The spectrum still retains the 60 Hz and 180 Hz energy associated with signal. The 60 Hz noise is suppressed by a factor of 20 and the 180 Hz noise is reduced by a factor of 5 in terms of total amplitudes. For comparison purposes, the data (figure 2a) were operated on by two notch filters (55-59-61-65 and 175-179-181-185) (figures 4a and 4b). A reflection event segregated by the window on figure 4a is less pronounce and does not appear to be coherent in comparison with the data processed by the hum filter (figure 2b). This difference results from the notch filters attenuating both power-lines noise as well as signal.

Some shot gathers from Zeandale possess only 60 Hz component power-line noise (figures 3a and 3c). After hum filtering the same shot gather, its spectrum (figure 3d) is testimony to the effectiveness of this technique. The 60 Hz noise is suppressed by a factor of 10 in amplitudes (figure 3b). For comparison, a notch filter was also applied to the data (figure 3a) with the results (figure 5a) similar to expected and observed in the previous example (figure 4a). Comparing events in the window on figures 3b and 5a, the hum filter clearly does a much better job of eliminating noise and retaining signal.

DISCUSSIONS AND CONCLUSIONS

Real world examples demonstrate the power of the hum filter as developed in this paper. This filter can eliminate harmonic noise such as power-line noise (60 Hz) and its multiples (120 Hz, 180 Hz, and 240 Hz etc.) on shallow seismic data without affecting the frequency content of signal. Real data examples also demonstrated the efficient and accurate filtering potential of this method when implemented in a normal shallow seismic data processing flow. Required computer time increases with the number of sinusoids of the power-line model (Eq. 1) and the length of the time window used to model power-line noise. For a 48-trace shot gather, less than 3 seconds on a Pentium-155 PC are needed for a model of three sinusoids such as 60 Hz, 120Hz, and 180 Hz with a length of 100 samples. The stability of the filter was tested by applying it to more than 6,000 shots of shallow seismic reflection data acquired near Zeandale, Kansas.

Because frequencies of the sinusoids are not specified in the power-line noise model (Eq. 1), this hum filter, can be used to remove harmonic noise with any frequency(ies).

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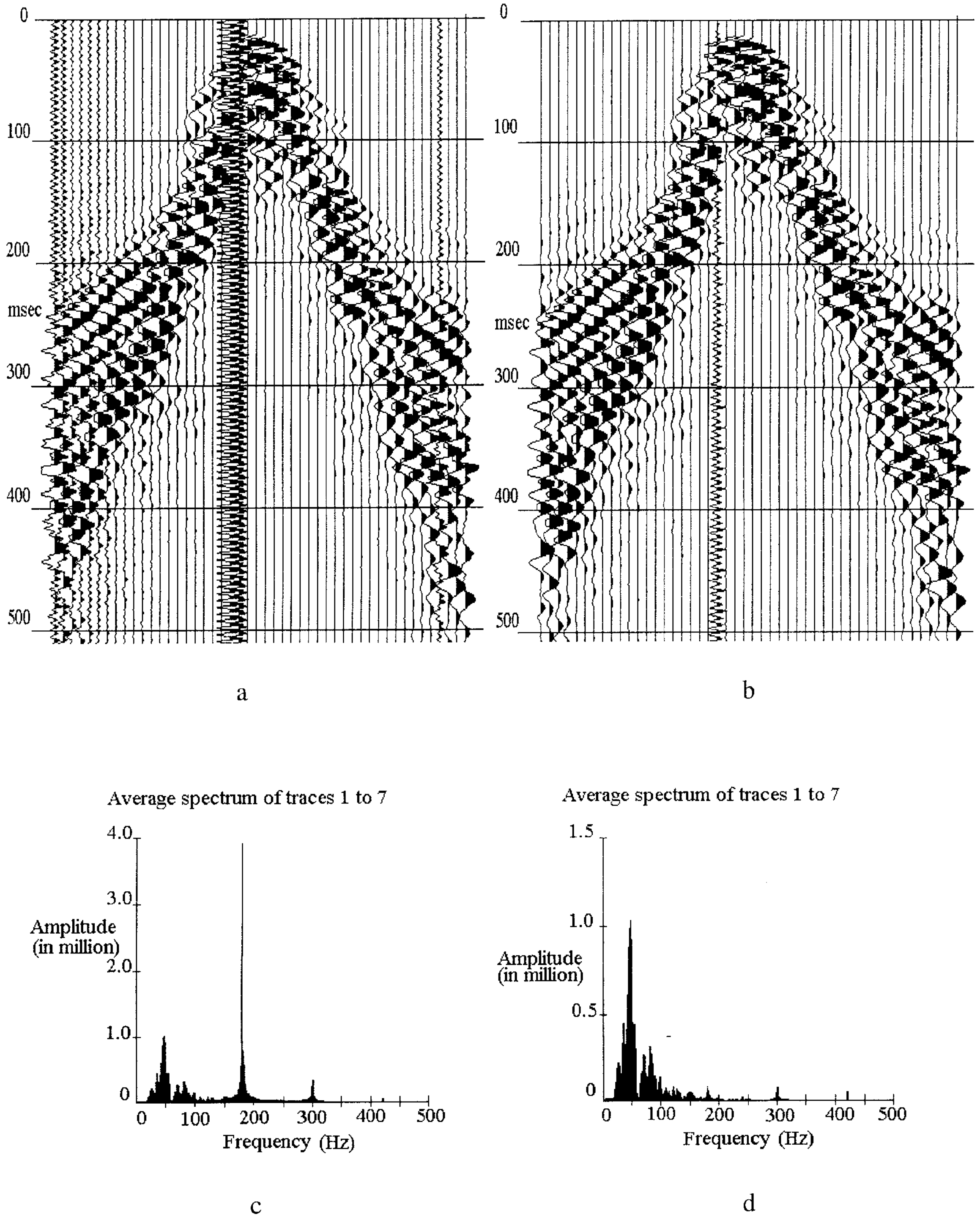


Figure 1. A 48-channel shot gather of seismic reflection data acquired at the Memphis Defense Depot, Memphis, Tennessee (a) and its spectrum (c); (b) the same shot gather after hum filtering (trace 21 was a dead trace) and the spectrum (d).

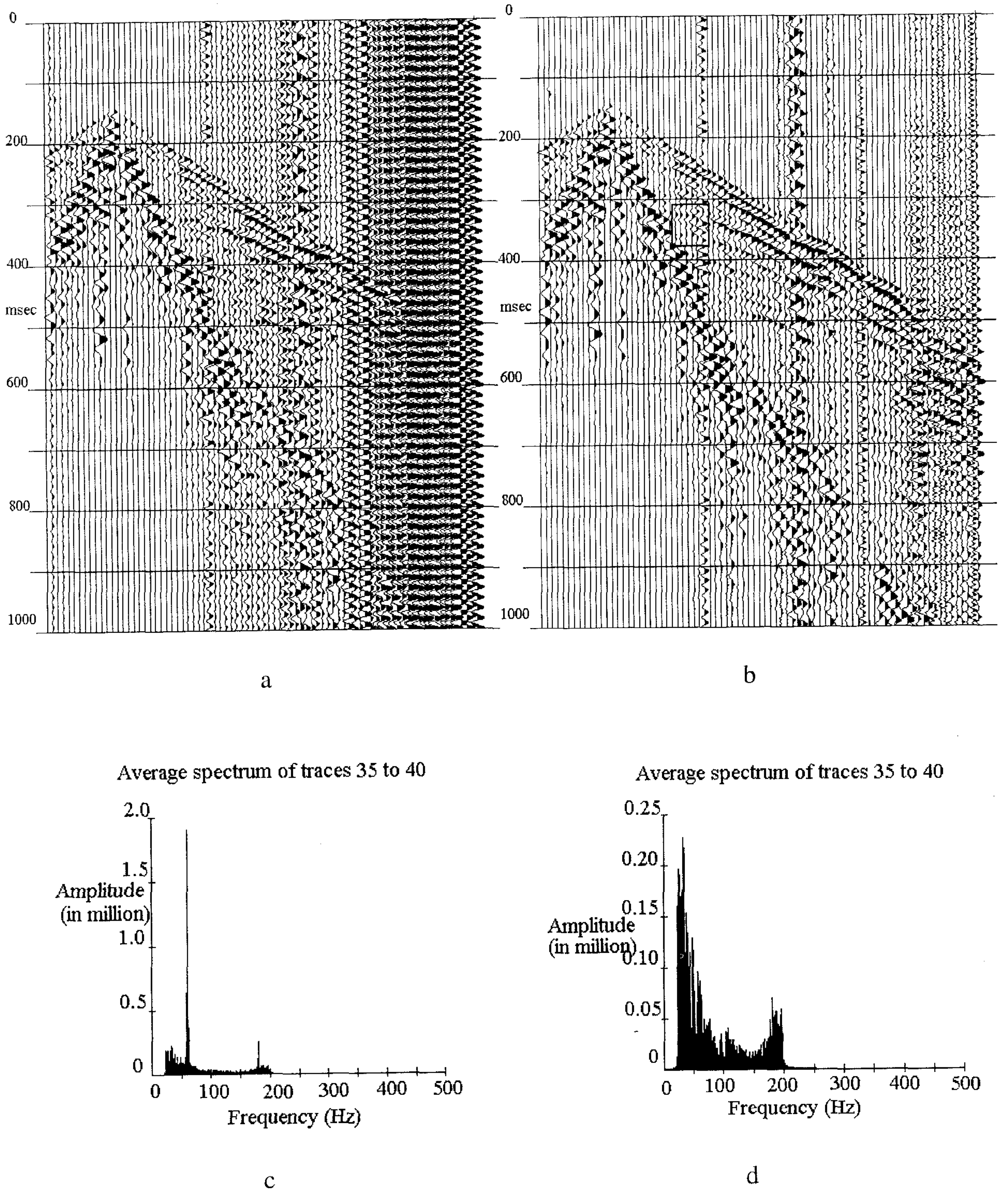


Figure 2. An 81-channel shot gather of seismic reflection data from Zeandale, Kansas with 60 Hz and 180 Hz power-line noise (a) and its spectrum (c); (b) the same shot gather after hum filtering and the spectrum (d).

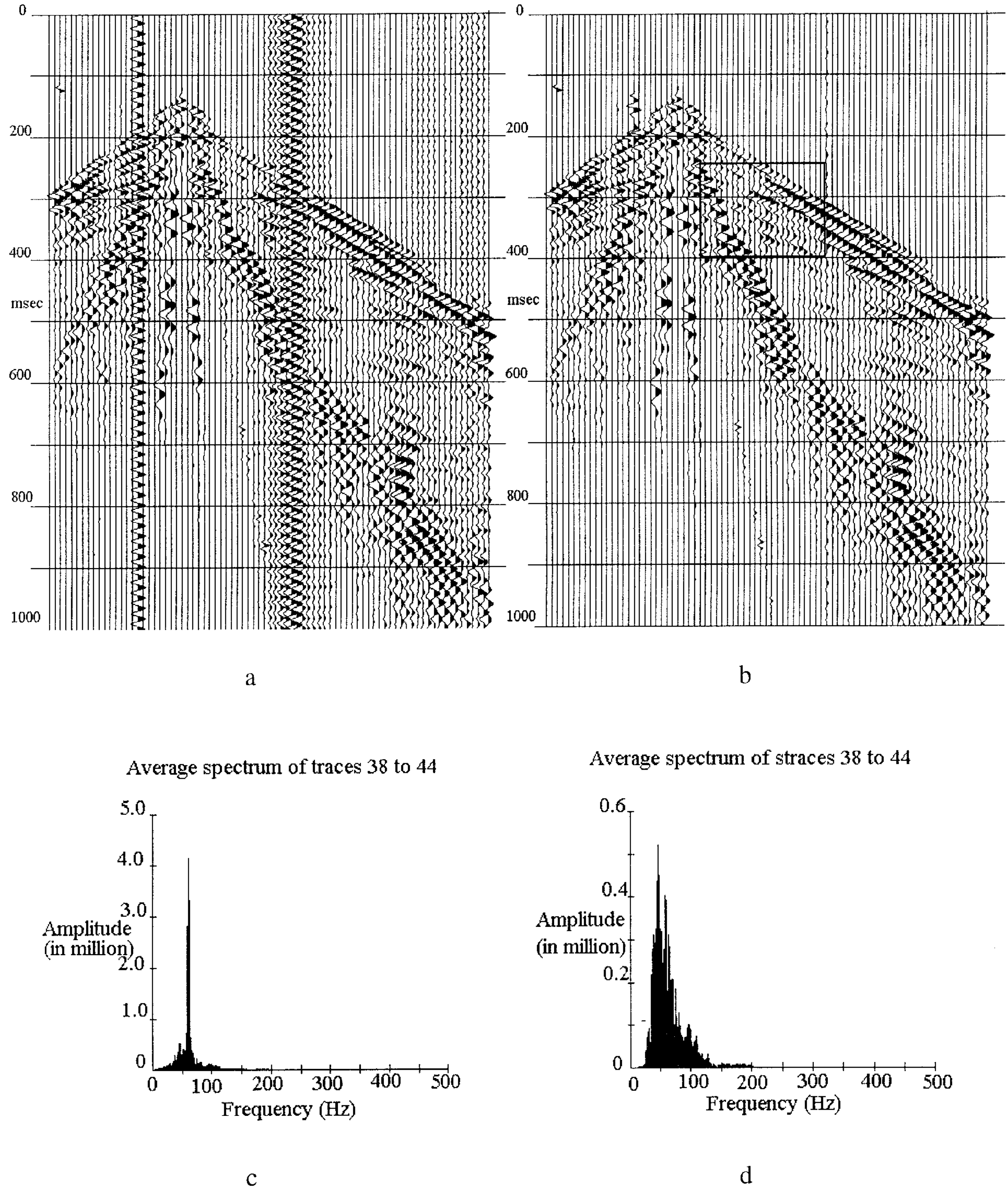
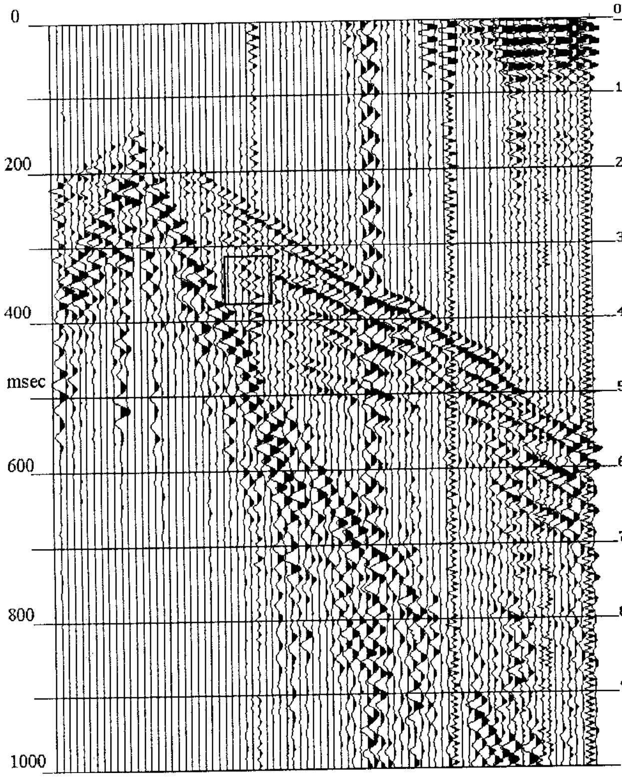


Figure 3. An 81-channel shot gather of seismic reflection data from Zeandale, Kansas with only 60 Hz power-line noise (a) and its spectrum (c); (b) the same shot gather after hum filtering and the spectrum (d).



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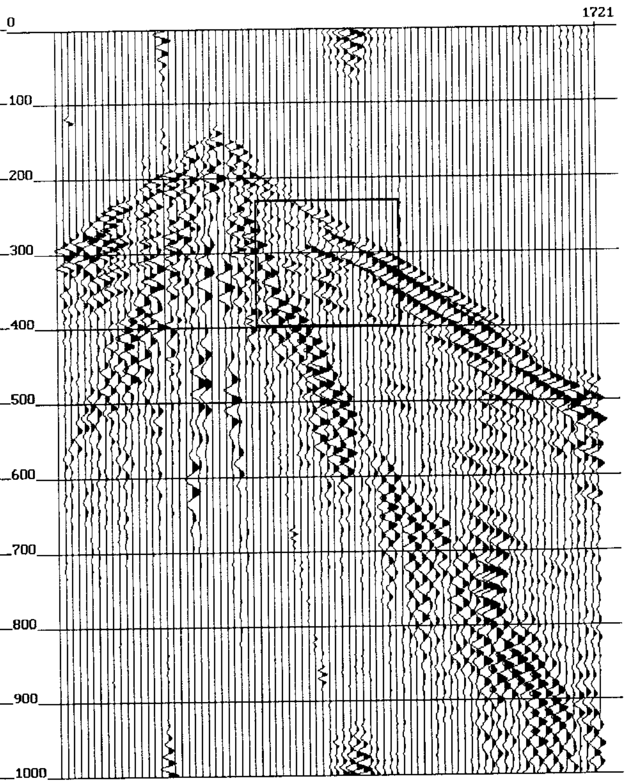

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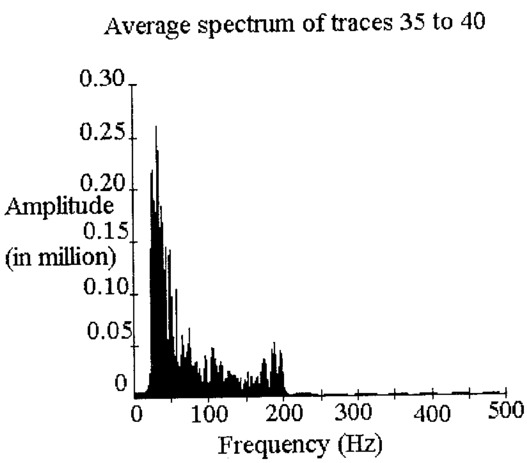

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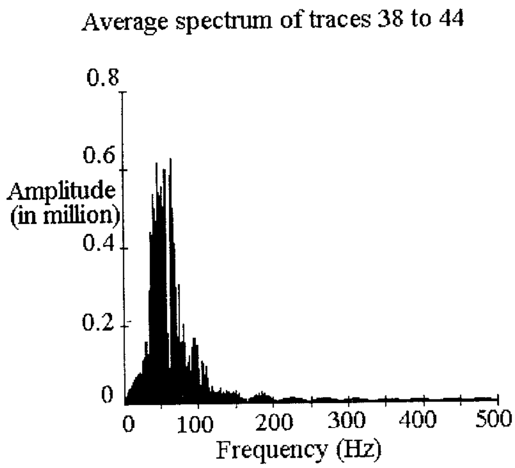

b

Figure 4. Notch filters applied to the data in figure 2a (a) and its spectrum (b).

Figure 5. A notch filter applied to the data in figure 3a (a) and its spectrum (b).