Correction of topographic distortions in potential-field data: A fast and accurate approach

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ABSTRACT
The equivalent source concept is used in the wave-number domain to correct distortions in potential-field data caused by topographic relief. The equivalent source distribution on a horizontal surface is determined iteratively through forward calculation of the anomaly on the topographic surface. Convergence of the solution is stable and rapid. The accuracy of the Fourier-based approach is demonstrated by two synthetic examples. For the gravity example, the rms error between the corrected anomaly and the desired anomaly is 0.01 mGal, which is less than 0.5 percent of the maximum synthetic anomaly. For the magnetic example, the rms error is 0.7 nT, which is less than 1 percent of the maximum synthetic anomaly. The efficiency of the approach is shown by application to the gravity and aeromagnetic grids for Kansas. For gravity data, with a maximum elevation change of 500 m reducing to a horizontal datum results in a maximum correction in gravity anomaly amplitude of up to 2.6 mGal. For aeromagnetic data, the method results in a maximum horizontal shift of anomalies of 470 m with a maximum correction in aeromagnetic anomaly amplitudes up to 270 nT.

INTRODUCTION
It is well known that topographic relief of the measurement surface causes distortion in potential-field data due to the varying vertical separation of the measurement points from the source bodies. Tsuboi (1965) refers to the Bouguer anomaly as a “station” Bouguer anomaly reduced to sea level and distinguishes it from the real Bouguer anomaly at sea level. The latter requires a vertical (upward and/or downward) continuation of the gravity field onto a common horizontal plane. Aeromagnetic data are usually measured on an irregular surface. The reduction of such data to some datum is necessary because most existing approaches for data enhancement and interpretation demand data on a horizontal plane.

Several methods have been proposed by previous workers to reduce observations to some datum. Dampney (1969) determined an equivalent source of discrete point masses on a horizontal plane from Bouguer anomaly measurements on an irregular surface by solving a system of simultaneous equations. By studying the condition number of the matrix of the system, he found that the appropriate depth to equivalent source is related to the station spacing. Henderson and Cordell (1971) discussed an approach to topographic correction by means of finite harmonic series. Bhattacharyya and Chan (1977) determined an equivalent source by solving a Fredholm integral equation of the second kind. Pilkington and Urquhart (1990) determined an equivalent source on a mirror image of the observation surface. When the mirror image surface is replaced by a horizontal plane, the anomaly caused by the equivalent source on the horizontal plane approximates the corrected anomaly on the corrected datum. Xia and Sprowl (1991) calculated a corrected anomaly from an ensemble of point-mass-equivalent sources, located at an optimum depth and determined from the iterative solution of the Dirichlet boundary-value problem. The optimum depth to the source ensemble is determined by maximizing the smoothness of the calculated anomaly between the data points. All of these approaches require significant computational time (except, Pilkington and Urquhart, 1990) when large data sets are handled.

In this study we present a fast and accurate method for determining an equivalent source for a large set of data measured on a topographic surface. Reduction to a horizontal plane is then straightforward.

THE METHOD
We define

\[ f(\mathbf{K}) = F[f(\mathbf{r})] \quad \text{and} \quad f(\mathbf{r}) = F^{-1}[f(\mathbf{K})], \]

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where $F$ and $F^{-1}$ are the Fourier transform and inverse Fourier transform of the function $f$, respectively. We use the right-hand coordinate system in the paper, in which the z-direction is positive downward. Considering the case in Figure 1, let $\sigma (K)$ be an equivalent density distribution on a given horizontal plane $E$, where $K = k_x e_x + k_y e_y$ is the wavevector, and $e_x$ and $e_y$ are unit vectors in the x- and y-directions, respectively. Our aim is to calculate the gravity anomaly on the observation surface $S$. The anomaly $\bar{g}(K)$ on the plane $E$ can be written as

$$\bar{g}(K) = 2\pi G \sigma (K),$$

(1)

where $G$ is the gravitational constant. Upward continuing $\bar{g}(K)$, we obtain the gravity anomaly on the observation surface $S$, $g(K)$, which is given by

$$g(K) = \bar{g}(K) \exp [-|K|Z(r)],$$

(2)

where $Z(r)$ is the vertical distance from the observation surface $S$ to the plane $E$, ($= x e_x + y e_y$) is the vector of coordinates in the $x$-$y$-plane.

If $Z(r)$ is constant, equation (2) reduces to the well-known upward continuation expression in the wavenumber domain. We will show that when $Z(r)$ is not constant, equation (2) can still be used to calculate $g(K)$ on the observation surface $S$, if the anomaly $\bar{g}(K)$ on the horizontal plane $E$ can be determined.

Let $Z_0$ be the median distance from the surface $Z(r)$ to the plane $E$, then

$$Z(r) = Z_0 + h(r),$$

(3)

where $h(r)$ is the topographic change relative to $Z_0$ (Figure 1). Equation (2) can be written as

$$g(K) = \bar{g}(K) \exp [-|K|Z_0] \exp [-|K|h(r)].$$

The exponential term in equation (2) is split into two exponential terms in equation (3). The first term is a conventional upward continuation operator, and the second term is related to topographic change. We use a Taylor series to express the term in brackets and substitute equation (1) for $\bar{g}(K)$. Equation (3) can then be written as

$$g(K) = 2\pi G \sigma (K) \exp [-|K|Z_0] \sum_{n=0}^{\infty} \frac{[-|K|h(r)]^n}{n!}.$$  

(4)

Applying the inverse Fourier transform to both sides of equation (4), we obtain,

$$g(r) = 2\pi G \sum_{n=0}^{\infty} \frac{h^n(r)}{n!} F^{-1}[\sigma(K) \exp (-|K|Z_0)] - K^n],$$

(5)

which is used later in an iterative process. Equation (5) is the basis for the reduction technique and was originally derived by Parker (1973), Guspì (1987), Pilkington and Urquhart (1990), and Pilkington (1990, personal communication) in different ways. The problem can then be solved if the series converges. Let $R = \max |\sigma (r)|$, then $|\sigma (K)| \leq RA$, where $A$ is the area covered by a data set, with $H = \max |h(r)|$, then

$$g(K) = 2\pi G \sigma (K) \exp [-|K|Z_0] \sum_{n=0}^{\infty} \frac{[-|K|h(r)]^n}{n!}$$

$$\leq 2\pi GAR \sum_{n=0}^{\infty} \frac{(H|K|)^n}{n!} \exp (-|K|Z_0)$$

$$= 2\pi GAR \sum_{n=0}^{\infty} L_n,$$

where

$$L_n = \frac{(H|K|)^n}{n!} \exp (-|K|Z_0) = \frac{(H|K|)^n}{n!} \sum_{j=0}^{\infty} \frac{(|K|Z_0)^j}{j!},$$

is independent of the wavenumber $K$. Because we can choose the plane $E$ below the observation surface $S$ ($Z_0 \geq H$), the series in equation (4) is uniformly convergent over the entire wavenumber domain, by the Weierstrass M-test (Whittaker and Watson 1962, p. 49), as is equation (5). Parker’s (1973) formula converges for the same reason.

Based on Poisson’s relation (e.g., Dobrin, 1975, p. 483), an equation for the magnetic anomaly can be written directly from equation (4), also in Parker (1973),

$$T(K) = 2\pi \frac{(\vec{K} \cdot f)(\vec{K} \cdot m)}{|K|^2} J(K)$$

(6)

Applying the inverse Fourier transform to both sides of equation (6), we obtain the formula used in the iterative process:

$$T(r) = 2\pi \sum_{n=0}^{\infty} \frac{h^n(r)}{n!} F^{-1}$$

$$\times \left[ \frac{(\vec{K} \cdot f)(\vec{K} \cdot m)}{|K|^2} J(K) \exp [-|K|Z_0] - K^n] \right],$$

(7)

where $\vec{K} = i(k_x e_x + k_y e_y) + \sqrt{k_x^2 + k_y^2} e_z$, $e_z$ is the unit vector in the z-direction and $i$ is the imaginary unit. Elements $f$ and $m$ are unit vectors with the direction of the earth’s field and the magnetization, respectively.
Equations (5) and (7) allow us to calculate gravity and magnetic anomalies on a topographic surface caused by a source located on a horizontal plane. We use the equations to determine an equivalent source distribution $\sigma(r)$ [or $J(r)$] on the plane $E$ based on measured anomalies on the observation surface $S$ by iterative forward calculations.

The procedure to determine the equivalent source distribution is described below.

1) Initialize $\sigma(r)$ [or $J(r)$], calculate $\sigma(K)$ [or $J(K)$], and define the depth to the equivalent source just below the lowest elevation of the observation surface.

2) Calculate the modeled gravity (or magnetic) anomaly from $\sigma(K)$ [or $J(K)$] by equation (5) [or (7)].

3) Estimate errors: two errors used to trace the iterative procedure are $\text{rms}$ error $\text{rms} (k)$ at the $k$th iteration:

$$\text{RMS} (k) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (s_i - u_i^k)^2}$$  \hspace{1cm} (8)

and the maximum deviation $\text{MAXD} (k)$ at the $k$th iteration:

$$\text{MAXD} (k) = \max_{1 \leq i \leq N} |s_i - u_i^k|,$$  \hspace{1cm} (9)

where superscript $k$ stands for the $k$th iteration and subscript $i$ for the $i$th data point; $s$ is the measured anomaly; $u$ is modeled anomaly calculated by equation (5) [or (7)], and $N$ is the total number of data points. If at any step, neither of these errors are reduced or the $\text{rms}$ error reaches the accuracy threshold, the iterative procedure is terminated.

4) Modify the $\sigma(r)$ [or $J(r)$] based on equation (10) (Bott, 1960), then return to step 2:

$$\sigma_i^{k+1} = \sigma_i^k + (s_i - g_i^k)/2\pi G$$

and

$$\text{(or) } J_i^{k+1} = J_i^k + C(s_i - T_i^k),$$

where $s$ is measured gravity (or magnetic) anomaly; $g$ and $T$ are calculated by equations (5) and (7), respectively; and $C$ is a dimensionless constant, which approximates to $1/2\pi$ when $J$ and $T$ are in the same units and chosen to produce convergence. In our experience, $C$ is chosen as 0.1. These simple equations of modification are effective and efficient.

Once the equivalent source on the plane $E$ is determined, the field on a horizontal plane (corrected datum) above the plane $E$ is given by the normal upward continuation in equation (2).

In this case, function $Z(r)$ is a constant, equal to the vertical distance from the datum to the plane $E$. The equivalent source can also be used to calculate pseudogravity, anomaly reduced to pole, directional directives, etc.

SYNTHETIC MODEL TESTING

The first example is from Xia and Sprowl (1991). A point-mass gravity source is buried 100 m directly beneath a 100 m vertical scarp at the surface. The source has an excess mass of $10^{10}$ kg. The data constitute a $15 \times 15$ grid of points, spaced every 100 m (Figure 2). Figure 3a shows the Bouguer anomaly measured on the topographic surface, after “normal” data corrections to a common datum have been performed. The distortion in the measured anomaly is due to the change in vertical separation between the source body and measuring stations. Figure 3b is the desired anomaly as would be measured on a horizontal datum 200 m above the source ($z = -100$ m; $z$ is positive downward). The initial equivalent density on the plane $z = 1$ m is set to zero and the gravity anomaly is calculated at each point on the measurement surface. The initial $\text{rms}$ and $\text{MAXD}$ errors are 0.316 mGal and 2.358 mGal. After 11 iterations, the $\text{rms}$ and $\text{MAXD}$ errors between the modeled and measured anomalies are reduced to 0.009 mGal and 0.126 mGal, respectively. The modified equivalent source is used to calculate the corrected anomaly on the plane $z = -100$ m, which is plotted in Figure 3c. The $\text{rms}$ error between the corrected anomaly (Figure 3c) and the desired anomaly (Figure 3b) is 0.012 mGal. The maximum and average values of correction of the example are 1.203 mGal and 0.059 mGal, respectively. Here, we define the value of correction as the difference between the measured data and the corrected data on a given horizontal plane. In this example, they are the difference between Figures 3a and 3c. Figure 3d shows the difference between Figures 3b and 3c demonstrating that the correction of the topographic distortion is very good.

The accuracy of the solution is not significantly dependent on the depth of the equivalent source. To confirm this, we set the equivalent source distribution at different depths.
from $z = 0.001$ m to 100 m for the example. The results show that the rms error between the corrected and true anomalies on the plane $z = -100$ m is in the region 0.011–0.012 mGal. However, the number of iterations increases from 7 to 53 with increasing depth to equivalent sources from 0.001 m to 100 m. Therefore, we usually define the equivalent source on the plane just below the lowest survey level to reduce the number of iterations in determining the values of the equivalent source distribution.

A second example uses a magnetized cube buried beneath a scarp with inclination = 60 degrees, declination = 30 degrees, and magnetization = $1/\pi \ A/m$ (= 400 nT). The solid cube is $100 \ m \times 100 \ m \times 100 \ m$, centered in the data area, with its top at a depth of 50 m. The data are a $15 \times 15$ grid of points, spaced every 100 m (Figure 4). Figure 5a shows the magnetic anomaly measured on the topographic surface. Figure 5b shows the desired anomaly as would be measured on a horizontal plane $z = -50$ m (half way up the scarp). The initial magnetization is set to zero and the depth to the equivalent sources is 1 m. The initial rms and MAXD errors are 11.618 nT and 76.580 nT, respectively. The rms and MAXD errors are reduced to 0.490 nT and 3.246 nT, respectively, after 42 iterations. Figure 5c shows the corrected anomaly on the plane $z = -50$ m. The rms error and average deviation between the corrected anomaly (Figure 5c) and the desired anomaly (Figure 5b) are 2.109 nT and 0.743 nT, respectively. Figure 5d plots the difference between Figures 5b and 5c. If the datum is chosen as $z = -100$ m, the rms error and average deviation between the corrected anomaly and desired anomaly are 0.520 nT and 0.250 nT, respectively.

**APPLICATION TO POTENTIAL-FIELD DATA IN KANSAS**

**Bouguer Gravity**

There are more than 59,000 gravity stations measured on the topographic surface in Kansas. The highest point on the topography is 1230 m above sea level in western Kansas and the lowest is 215 m above sea level in eastern Kansas. The kriging method of SURFACE III (Sampson, 1988) is used to grid Bouguer gravity data to $1.6 \times 1.6$ km. The final gridded data set is $205 \times 408$ points. The original Bouguer anomaly map is shown in Figure 6.

The initial equivalent density is 0 g/cm$^3$ and the equivalent source is on a plane $z = -200$ m (200 m above sea level), just below the lowest measurement point. The initial rms and MAXD errors are 79.9 mGal and 151.1 mGal, respectively. The rms and MAXD errors are reduced to 0.1 mGal and

![Fig. 3. Results for the point-mass example. (a) Bouguer anomaly on the topographic surface; (b) expected anomaly on the plane $z = -100$ m; (c) topographically corrected anomaly on the plane $z = -100$ m; (d) difference between (b) and (c). The units in both x- and y-directions are one station spacing, 100 m. Contour interval is 0.2 mGal in (a)–(c), and 0.02 mGal in (d).](image-url)
Correction of Potential-Field Data

Aeromagnetic Anomaly

There are about 72,000 line-km (8–11 measurements/km) of aeromagnetic data in Kansas. The average distance between flight lines is 3.2 km. The data were measured at three different elevations, 762.0 m (2500 ft) above sea level over the eastern half of Kansas, 914.4 m (3000 ft) over the west-central quarter, and 1371.6 m (4500 ft) above sea level over the western quarter of the state. There is a transition zone about 5–15 km wide in western Kansas, over which the plane changed elevation from 914.4 m to 1371.6 m. The reductions took about 4 minutes on a Data General MV 20,000.

\[ \text{Datum E. Kansas correction} \]

<table>
<thead>
<tr>
<th>Datum (m)</th>
<th>Western Kansas</th>
<th>E. Kansas</th>
<th>Maximum correction (nT)</th>
</tr>
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<td>14</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
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<td>11</td>
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<td>5</td>
</tr>
<tr>
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<td>3</td>
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<td>14</td>
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The topographic and flight-line elevation distortions to the gravity and magnetic data of Kansas are significant but are readily corrected using the method developed in this paper. The modest topographic relief of Kansas suggests that topographic distortions should not be ignored in the common study of gravity and magnetic data. We suggest that this method, or an equivalent one, be incorporated into the data reduction stream in future gravity and magnetic studies. Topographic corrections are not as trivial as the fundamental corrections for latitude and elevation and must remain under the control of the investigator, but the method developed above is objective, stable, and rapid, and routine application is not burdensome.
FIG. 5. Results of the rectangular solid example. (a) Magnetic anomaly on the measurement surface; (b) expected anomaly on the plane $z = -50$ m; (c) topographically corrected anomaly on the plane $z = -50$ m. (d) difference between (b) and (c). The units in both $x$- and $y$-directions are one station spacing, 100 m. Contour interval is 10 nT in (a)–(c), 5 nT in (d).

FIG. 6. Bouguer gravity anomaly of Kansas measured on the topographic surface. Coordinates in $x$- and $y$-directions are longitude and latitude in degrees. Contour interval is 4 mGal.
FIG. 7. Corrected Bouguer gravity of Kansas on the plane 700 m above sea level. Coordinates in x- and y-directions are longitude and latitude in degrees. Contour interval is 4 mGal.

FIG. 8. Values of the topographic correction, which is the difference between Bouguer gravity on the measurement surface (Figure 6) and the corrected anomaly (Figure 7). The Coordinates in x- and y-directions are longitude and latitude in degrees. Shading interval is 0.2 mGal.
FIG. 9. Aeromagnetic anomaly of Kansas measured at three different elevations. Coordinates in x- and y-directions are longitude and latitude in degrees. Contour interval is 100 nT.

FIG. 10. Corrected aeromagnetic anomaly of Kansas on the datum 914.4 m above sea level. Coordinates in x- and y-directions are longitude and latitude in degrees. Contour interval is 100 nT.
Fig. 11. Values of the topographic correction, which is the difference between the aeromagnetic anomaly on the three different levels (Figure 9) and the corrected anomaly on the plane 914.4 m above sea level (Figure 10). Coordinates in x- and y-directions are longitude and latitude in degrees. Shading interval is 10 nT.

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