

SUMMARY

We have derived an analytical expression in the wavenumber domain for calculating the gravity anomaly caused by an exponential density contrast model for an interface (Xia and Sprowl, 1990), above which the density contrast changes exponentially with depth. Constant density contrast and a linear density contrast are specific cases of the exponential density contrast model. The formula should be more efficient than vertical prism methods for linear or exponential density contrast. The formula is tested using synthetic data. We employ the approach presented by Xia and Sprowl (1992) to invert the regional gravity anomaly of Kansas to determine depth to the Moho discontinuity with an exponential density contrast model. The inversion is constrained by seismic refraction along the two-dimensional profile from Concordia, Kansas to Agate, Colorado (Steeple and Miller, 1989). The average density model in the crust of Kansas is also determined.

INTRODUCTION

In the mapping of a density interface, complex geology requires consideration of a variable density contrast above the interface as a function of depth. It is well known that the density of sedimentary rocks increases with depth of burial (Athy, 1930). The density of the rocks rapidly increases at shallow depths and increases progressively less rapidly at greater depths. An exponential density contrast model is a reasonable approximation of a sedimentary boundary and is significantly more appropriate than a constant density contrast model or a linear density contrast model.

It is difficult, however, to derive formulas for calculating gravity anomalies due to depth-dependent density in the spatial domain. Reamer (1986) derived a formula for the gravity anomaly of a linear density contrast model in the wavenumber domain. Reamer and Ferguson (1989) also discussed an inversion of gravity anomaly into a density interface for the linear density contrast model in the wavenumber domain. It is impossible to obtain an analytical expression in the spatial domain for the gravity anomaly of even a simple geometrical body when the density of the body varies exponentially with depth (Murthy and Rao; 1979). Cordell (1973) proposed a recursive procedure and Murthy and Rao (1979) extended Hubbert's (1948) line-integral method for the case of a linear density model to obtain an approximate solution for the exponential density contrast model. Chai and Hinze (1988) and Chenot and Debeglia (1990) presented an approach based on using vertical prisms with the exponential density contrast model in the wavenumber domain to invert gravity anomaly into density interface.

In this paper, we first derive an analytical expression in the wavenumber domain for calculating the gravity anomaly produced by an exponential density contrast model for an interface, above which density contrast changes with depth exponentially. In order to show that the analytical formula of the exponential density contrast model is correct, we calculate the anomalies caused by a set of rectangles with varying densities, which are determined by an exponential model. The summation of all anomalies caused by the rectangles should be very close to the anomaly directly calculated by the formula of the exponential density contrast model. Finally, we invert the gravity anomaly by iterative forward modeling (Xia and Sprowl, 1992) with the exponential density contrast model and apply this approach to

determine the depth to the Moho discontinuity under the state of Kansas.

FORWARD FORMULA OF AN EXPONENTIAL DENSITY CONTRAST MODEL

The exponential density contrast model is

$$\Delta\rho(z) = a + be^{-\mu z} \tag{1}$$

where $(a+b)$ is the density contrast between surface rocks and rocks below a density interface and μ is a decay constant with the unit of $1/\text{length}$. The gravity anomaly caused by the density interface with this density model is $h = h_1 + h_2$. The first term h_1 , which is caused by the constant density a , can be calculated by Parker's formula (1973). We have derived a formula (Equation 2) for calculating the gravity anomaly h_2 caused by the second exponential term (see Appendix).

$$F[h_2(x, y)] = 2\pi G b \sum_{n=1}^{\infty} \frac{[-(\mu + |\bar{k}|)]^{n-1}}{n!} \times \left\{ e^{-(\mu + |\bar{k}|)\delta_2} F[(z_2(x, y) - \delta_2)^n] - e^{-(\mu + |\bar{k}|)\delta_1} F[(z_1(x, y) - \delta_1)^n] \right\} \tag{2}$$

where F is Fourier transform; G is the gravitational constant; \bar{k} is the wavenumber vector, $\bar{k} = k_x \bar{e}_x + k_y \bar{e}_y$, \bar{e}_x and \bar{e}_y are the unit vectors in x and y directions, respectively; $z_1(x, y)$ (ZT) and $z_2(x, y)$ (ZB) are the top surface and the bottom surface of the layer, respectively; δ_1 and δ_2 are the median values of $z_1(x, y)$ and $z_2(x, y)$, respectively.

If μ is zero, the second term in Equation (1) become the constant b , and Equation (2) turns out to be Parker's formula as expected. We have shown the linear density contrast model (Reamer, 1986) is a specific case of the exponential density contrast model. A linear density model used by Reamer and Ferguson (1989), for example,

$\rho(z) = -750 + 100z \text{ kg/m}^3$, $0 \leq z \leq 4 \text{ km}$, could be approximately expressed by an exponential density contrast model

$$\rho(z) = 99,250 - 100,000e^{-0.001z}$$

It is also apparent that calculation of h_2 will require the same order of magnitude of time as the calculation of h_1 . Equation (2) should be more computationally efficient than the formula which is based on using vertical prisms with the exponential model (Equation 1) to fit an interface (Chai and Hinze, 1988; Chenot and Debeglia 1990). The reason for this time consumed by Fourier transformation in solving Equation (2) is much less than that of the approaches of Chai and Hinze (1988) and Chenot and Debeglia (1990).

TESTING BY A RECTANGULAR SOLID MODEL

We choose a rectangular solid of dimensions $1,000 \text{ m} \times 1,000 \text{ m} \times 800 \text{ m}$ ($3,300 \text{ ft} \times 3,300 \text{ ft} \times 2,600 \text{ ft}$) with a density contrast

$$\Delta\rho(z) = 0 \text{ g/cm}^3, \quad 0 \text{ m} < z < 50 \text{ m}, \quad (3a)$$

$$\Delta\rho(z) = -0.5e^{-0.0018z} \text{ g/cm}^3, \quad 50 \text{ m} \leq z \leq 850 \text{ m}. \quad (3b)$$

It is located in the center of the data area with top at a depth of 50 m (160 ft) and bottom at a depth of 850 m (2,800 ft). The data set is 50×50 with a station spacing of 100 m in both x and y directions.

Figure 1 shows the solution of Equation (2) with the density contrast model of Equation (3). The series in Equation (2) are cut off after n is larger than 8. The ratio of value of the eighth term to the summation of first seven terms is less than 1.5×10^{-6} , demonstrating rapid convergence of Equation (2).

We divide the rectangular solid into eight thinner rectangles, each of which has thickness of 100 m (330 ft). The top of first thinner rectangle is 50 m (165 ft) and its bottom at the depth of 150 m (495 ft), the top of the second thinner rectangle is 150 m (495 ft) and its bottom at the depth of 250 m (825 ft), and so on. The values of density contrast for each rectangle from top to bottom are -0.457, -0.382, -0.319, -0.266, -0.222, -0.186, -0.155, and -0.130 g/cm^3 , respectively. The anomalies caused by all these thinner rectangles are calculated by a rectangular formula (Nagy, 1966). Figure 2 shows the difference between Figure 1 and the summation of anomalies caused by all these thinner rectangles.

The maximum deviation between both anomalies is 0.25 mGal, which is less than 5 percent of the amplitude of the expected anomaly. The difference in the two anomalies is caused mainly by noise of the Gibbs phenomena in Fourier transformation.

INVERSE APPROACH

Two main steps in the iterative forward-modeling approach are to determine an initial model and to modify the model (Xia and Sprowl, 1992).

Initial model. We initialize ZT (the depth to the top of the layer) to an average depth and define an exponential density contrast model and the depth to the bottom of the layer (ZB), which are kept unchanged while solving for ZT . After these three parameters are determined, we can calculate the anomaly due to the initial model with Equation (2).

Model modification. The formula used to modify the depth to the density interface after each iteration is shown in Equation (4) (Bott, 1960),

$$\Delta h_i^k = (g_i - h_i^k) / 2\pi G \Delta\rho_i, \quad (4)$$

where superscript k is the k th iteration and subscript i is the i th data point; Δh_i^k is the modification to the depth ZT_i^k , the depth of ZT below point i at the k th iteration; g_i and h_i^k are the measured and calculated gravity anomalies, respectively; G is the gravitational constant; and $\Delta\rho_i$ is the density contrast, which is function of the depth. Bott (1960) used Equation (4) to modify a two-dimensional gravity model.

Two error parameters used to measure the effectiveness of the iterative procedure are a root-mean-square error, $RMS(k)$, at the k th iteration,

$$RMS(k) = \sqrt{\frac{1}{N} \sum_{i=1}^N (h_i^k - g_i)^2} \quad (5)$$

and the maximum deviation, $MAXD(k)$, at the k th iteration,

$$MAXD(k) = \max_{1 \leq i \leq N} |h_i^k - g_i| \quad (6)$$

where N is the total number of data points.

The inverse approach is described by Xia and Sprowl (1992). Iterative improvement in the depth to the interface should cause reduction in both of these errors, and the errors are minimized as the solution converges. In practice, a given iteration may not reduce both errors simultaneously because the modifications to the model are approximate. Iteration continues

until no further improvement in either error is realized or when RMS reaches the accuracy threshold of the observational data.

The uniqueness of the derived model of depth to the top of the layer is also restricted by the uncertainty in the assumed exponential density contrast model and the assumed average depth to this interface. Clearly, reduction in the assumed exponential density contrast (or increase in average depth to interface) increases topographic relief on the calculated surface. Appropriate solutions are possible only when the initial parameters are constrained by other geological or geophysical information.

GEOLOGICAL EXAMPLE

The regional gravity anomaly, which contains 408 by 205 grid points, is the second order trend of the Bouguer anomaly for the state of Kansas and is assumed to be due to the Moho discontinuity (Xia, 1992). We try to invert the regional gravity to derive the depth to the Moho discontinuity by the approach discussed in above section. We assume an exponential density contrast for the crust. The inverse result is constrained by seismic refraction data (Steeple and Miller, 1989). We use the trial-and-error method to determine the average depth to the Moho and the exponential density contrast model. The criterion is that the determined exponential density contrast model and the average depth to the Moho should produce the best coincidence between the gravity inversion and the seismic data along the refraction profile.

We finally determined the average depth to the Moho as 37.0 km and the exponential density contrast model as

$$\Delta\rho(z) = -1.0e^{-0.0187z} \text{ g/cm}^3, \quad (7)$$

where the unit of depth z is km. Thus, the density contrast with the mantle is -1.0 g/cm^3 at the surface and -0.50 g/cm^3 at the average depth of the Moho. This suggests a surface density of 2.35 g/cm^3 and a density of 2.85 g/cm^3 at depth of 37.0 km (23.0 mi). The average density contrast can be determined by

$$\Delta\bar{\rho} = \frac{1}{37.0} \int_0^{37.0} -1.0e^{-0.0187z} dz = -0.72 \text{ g/cm}^3. \quad (8)$$

Therefore, the average density of the crust is $3.35 - 0.72 = 2.63 \text{ g/cm}^3$, which is less than the conventional density for continental crust of 2.67 g/cm^3 . The density values are geologically reasonable.

We defined the lower interface $ZB = 100$ km. An initial guess for ZT is 37.0 km. The initial errors are $RMS = 25.7$ mGal and $MAXD = 76.8$ mGal. Five iterations reduce the errors to $RMS = 0.8$ mGal (1.0% of the maximum real anomaly) and $MAXD = 11.5$ mGal. The calculations took 824 CPU seconds on a Data General MV20000 (for constant density model, 529 CPU seconds are needed in three iterations. CPU time used per iteration is largely model independent). The final result, which represents the depth to a smoothed surface of the Moho, is shown in Figure 3. The gravity inversion for the depth to the Moho at Concordia, Kansas is 35.2 km (21.9 mi), which is 0.8 km shallower than the result from seismic refraction. The inverted depth to the Moho at the intersection point of the Kansas - Colorado boundary and the seismic refraction profile is 43.5 km (27.0 mi), which is about 1 km deeper than the result from seismic refraction. The thinner crust in the area around 97.2 degrees of the longitude and 39.2 degrees of the latitude is the area postulated by Miller (1983), as high-velocity crust. Both gravity and seismic data suggest the presence of mantle material at fairly shallow depths in this region of the CNARS. The inverse result from gravity data looks reasonable, but there clearly is some edge distortion caused by the Gibbs phenomenon of Fourier transform.

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APPENDIX. Derivation of the Formula Calculating Gravity Anomaly due to an Exponential Density Contrast Model

The exponential density model is

$$\rho(z) = a + be^{-\mu z} \tag{A.1}$$

The gravity effect due to this density model is given by

$$h = h_1 + h_2, \tag{A.2}$$

where the first term h_1 , which is caused by a constant contrast density model a , can be calculated by Parker's formula. In the following paragraphs, we will derive the formula for calculating the second term h_2 , which is caused by the exponential density contrast model.

The gravity anomaly caused by a single-point mass of a body $m(\vec{r}_0, z_0)$ located at (\vec{r}_0, z_0) (\vec{r}_0 is the horizontal position vector, $\vec{r}_0 = x_0\vec{e}_x + y_0\vec{e}_y$, \vec{e}_x and \vec{e}_y are the unit vectors in x and y directions, respectively; z_0 is the vertical coordinate of the location of the point-mass).

$$dh_2(\vec{r}) = m(\vec{r}_0, z_0)g_d(\vec{r} - \vec{r}_0, z_0)dx_0dy_0dz_0,$$

where $g_d(\vec{r}, z_0)$ is the anomaly of unit mass, $\vec{r} = x\vec{e}_x + y\vec{e}_y$. Integration over the limits of the body yields the anomaly caused by the entire body:

$$h_2(\vec{r}) = \int_{z_1(\vec{r}_0)}^{z_2(\vec{r}_0)} \iint m(\vec{r}_0, z_0)g_d(\vec{r} - \vec{r}_0, z_0)dx_0dy_0dz_0. \tag{A.3}$$

Fourier transformation of both sides of Equation (A.3) and reordering yields

$$\begin{aligned} F[h_2(\vec{r})] &= \iint_{\vec{r}_0} \int_{z_1(\vec{r}_0)}^{z_2(\vec{r}_0)} m(\vec{r}_0, z_0)F[g_d(\vec{r} - \vec{r}_0, z_0)]dz_0dx_0dy_0 \\ &= \iint_{\vec{r}_0} \int_{z_1(\vec{r}_0)}^{z_2(\vec{r}_0)} m(\vec{r}_0, z_0)e^{-i\vec{k}\cdot\vec{r}_0}F[g_d(\vec{r}, z_0)]dz_0dx_0dy_0, \end{aligned} \tag{A.4}$$

where

$$F[g_d(\vec{r}, z_0)] = 2\pi G e^{-|\vec{k}|z_0} \tag{A.5a}$$

is the expression of the single-point mass in the wavenumber domain; G is a gravitational constant; \vec{k} is the wavenumber vector ($\vec{k} = k_x\vec{e}_x + k_y\vec{e}_y$); $z_1(\vec{r}_0)$ and $z_2(\vec{r}_0)$ are the top surface and the bottom surface of the layer, respectively; and

$$m(\vec{r}_0, z_0) = be^{-\mu z_0} \tag{A.5b}$$

Substituting Equations (A.5) into Equation (A.4) and performing the integration over z_0 yields

$$\begin{aligned} F[h(\vec{r})] &= \frac{-2\pi G b}{(\mu + |\vec{k}|)} \iint_{\vec{r}_0} \left[e^{-(\mu + |\vec{k}|)z_2(\vec{r}_0)} - e^{-(\mu + |\vec{k}|)z_1(\vec{r}_0)} \right] e^{-i\vec{k}\cdot\vec{r}_0} dx_0 dy_0 \\ &= \frac{-2\pi G b}{(\mu + |\vec{k}|)} F \left[e^{-(\mu + |\vec{k}|)z_2(\vec{r}_0)} - e^{-(\mu + |\vec{k}|)z_1(\vec{r}_0)} \right]. \end{aligned} \tag{A.6}$$

Let δ_1 and δ_2 be the median values of $z_1(\vec{r}_0)$ and $z_2(\vec{r}_0)$, respectively, then Equation (C.6) becomes

$$\begin{aligned} F[h_2(\vec{r})] &= \frac{-2\pi G b}{(\mu + |\vec{k}|)} \times \\ &\left\{ e^{-(\mu + |\vec{k}|)\delta_2} F \left[e^{-(\mu + |\vec{k}|)(z_2(\vec{r}_0) - \delta_2)} \right] - \right. \\ &\left. e^{-(\mu + |\vec{k}|)\delta_1} F \left[e^{-(\mu + |\vec{k}|)(z_1(\vec{r}_0) - \delta_1)} \right] \right\}. \end{aligned}$$

Now replacing the two inner exponential terms by power series and performing some minor simplification yields the final equation

$$\begin{aligned} F[h_2(\vec{r})] &= 2\pi G b \sum_{n=1}^{\infty} \frac{\left[-(\mu + |\vec{k}|) \right]^{n-1}}{n!} \times \\ &\left\{ e^{-(\mu + |\vec{k}|)\delta_2} F \left[(z_2(\vec{r}_0) - \delta_2)^n \right] - e^{-(\mu + |\vec{k}|)\delta_1} F \left[(z_1(\vec{r}_0) - \delta_1)^n \right] \right\}. \end{aligned}$$

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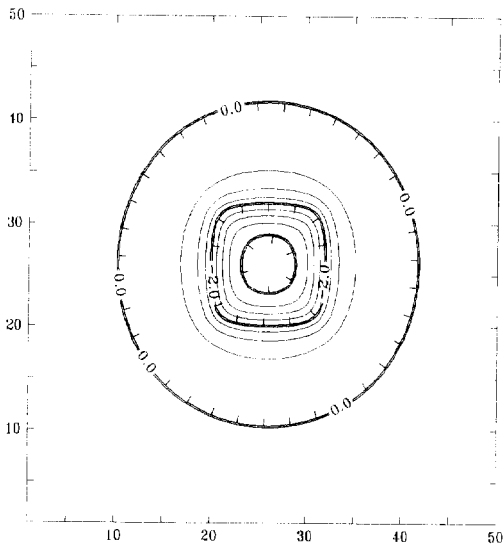


Figure 1. The gravity anomaly caused by the rectangular solid model with the density model shown in Equation (3), calculated by Equation (2). Contour interval is 0.5 mGal. The unit in both x and y directions is one station spacing, 100 m.

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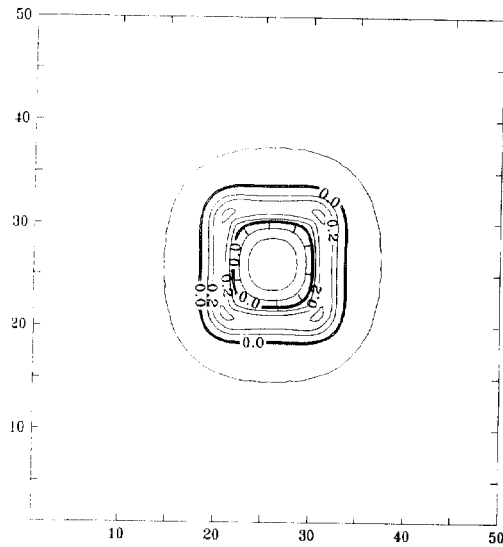


Figure 2. The difference between the gravity anomaly shown in Figure 1 and the summation of gravity anomalies caused by the eight thinner rectangular solids with the density model shown in Equation (3) and calculated by the rectangular formula (Nagy, 1966). Contour interval is 0.1 mGal. The unit in both x and y directions is one station spacing, 100 m.

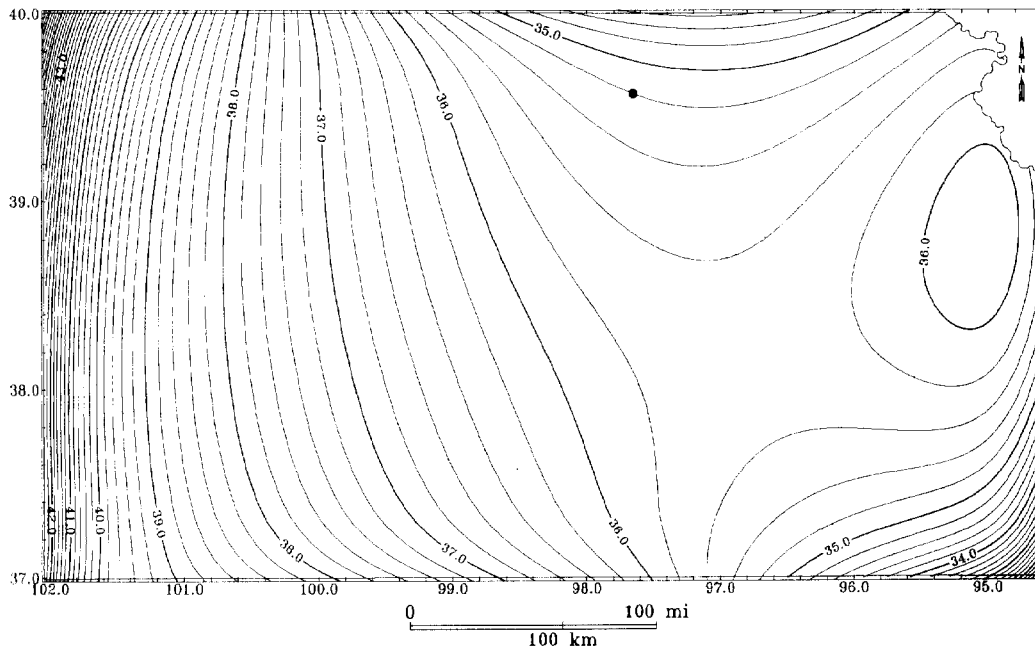


Figure 3. The depth to the Moho discontinuity calculated by inversion of Kansas gravity data with the exponential density contrast model shown in Equation (7). The black spot shows the location of Concordia, Kansas. Contour interval is 0.2 km. Datum is sea level. Coordinates in x and y directions are degrees of longitude and latitude, respectively.