Gravity and Magnetics 2

Tuesday Afternoon, November 12th

A Fast and Accurate Approach: Correction of Topographic Distortions in Potential-Field Data

G/M2.1

Jianghai Xia, Kansas Geological Survey; Donald R. Sprowl, Univ. of Kansas; and Dana Adkins-Heljeson, Interactive Concepts Inc.*

SUMMARY

The numerical equivalent source is used in the wavenumber domain to correct distortions in potential-field data caused by topographic relief. The equivalent source is determined iteratively by an accurate forward formula. Convergence of the solution is stable and rapid. The approach is verified with two synthetic examples and then applied to the gravity and aeromagnetic grids for Kansas, each of which consists of 205 × 408 points.

INTRODUCTION

It is well known that topographic relief at the surface of measurement causes distortion of potential-field data due to the varying vertical separation of the measurement points from the source body. Tsuboi (1965) refers to the Bouguer anomaly as a "station" Bouguer anomaly reduced to sea level and distinguishes it from the real Bouguer anomaly at sea level. The latter requires a vertical (upward and/or downward) continuation of the gravity field onto a common horizontal plane.

Various methods have been used for distortion reduction. Dampney (1969) determined an equivalent source of discrete point masses on a horizontal plane from Bouguer anomaly measurements on an irregular surface by solving a system of simultaneous equations. By studying the condition number of the matrix of the system, he found that the appropriate depth to equivalent source is related to the station spacing. Henderson and Cordell (1971) discussed an approach of topographic correction by means of finite harmonic series. Syberg (1972) developed continuation operators, which was a two-dimensional integral in the wavenumber domain, for reducing potential field data from a general surface to another general surface. Bhattacharyya and Chan (1977) determined an equivalent source by solving a Fredholm integral equation of the second kind. Pilkington and Urquhart (1990) determined an equivalent source on a mirror image of the observation surface. This mirror image surface is then replaced by a horizontal plane and the corrected anomaly on the corrected datum approximates the anomaly caused by the equivalent source on the horizontal plane. Xia and Sprowl (1991) calculated a corrected anomaly from an ensemble of point-mass-equivalent sources, located at an optimum depth, and derived from the iterative solution of the Dirichlet boundary-value problem. The optimum depth to the source ensemble is determined by maximizing the smoothness of the calculated anomaly between the data points. All of these approaches require significant computational time (except Pilkington and Urquhart (1990)) when large data sets are handled.

In this study we present a fast and accurate method for determining an equivalent source for a large data set measured on a topographic surface. Reduction to a horizontal plane is then straight-forward.

THE METHOD

We define

$$f(\vec{K}) = F[f(\vec{r})] \text{ and } f(\vec{r}) = F^{-1}[f(\vec{K})],$$

where F and F^{-1} are Fourier transform and inverse Fourier transform of function f, respectively. Considering the case in Figure 1, let $\bar{g}(\vec{K})$ be a gravity anomaly on a given horizontal plane E, where $\vec{K}\,(= k_x\vec{e}_x + k_y\vec{e}_y)$ is the wave vector, and $\vec{e}_x$ and $\vec{e}_y$ are the unit vectors in x and y directions, respectively. The anomaly can be written as

$$\bar{g}(\vec{K}) = 2\pi G\sigma(\vec{K}), \tag{1}$$

where G is gravitational constant, $\sigma(\vec{K})$ is an equivalent source on the plane E. To apply upward continuation to $\bar{g}(\vec{K})$, we can write

$$g(\vec{K}) = \bar{g}(\vec{K})\exp[-|\vec{K}|Z(\vec{r})], \tag{2}$$

where $Z(\vec{r})$ is the vertical distance from observation surface S to the plane E, (see Figure 1), and $\vec{r}\,(= x\vec{e}_x + y\vec{e}_y)$ is the vector of coordinates on the x-y plane.

If $Z(\vec{r})$ is constant, equation (2) is a well-known upward continuation expression in the wavenumber domain. We show that equation (2) can still be used to calculate $g(\vec{K})$ on the observation surface S, when $Z(\vec{r})$ is not constant, if the anomaly $\bar{g}(\vec{K})$ on the horizontal plane E can be determined.

Let Z_0 be the median distance from the surface $Z(\vec{r})$ to the plane E, then

$$Z(\vec{r}) = h(\vec{r}) + Z_0,$$

where $h(\vec{r})$ is the topographic change relative to Z_0. Equation (2) can be written as

$$g(\vec{K}) = \bar{g}(\vec{K})\exp[-|\vec{K}|Z_0]\{\exp[-|\vec{K}|h(\vec{r})]\}. \tag{3}$$

We use a Taylor series to express the term in { } and substitute equation (1) for $\bar{g}(\vec{K})$, then equation (3) can be written as

$$g(\vec{K}) = 2\pi G\sigma(\vec{K})\exp[-|\vec{K}|Z_0]\sum_{n=0}^{\infty}\frac{[-|\vec{K}|h(\vec{r})]^n}{n!}. \tag{4}$$

Equation (4) is the basis for the reduction technique, which is also mentioned by Parker (1973), Gusip (1987), Pilkington and Urquhart (1990), and Pilkington (1990) in different ways. The problem can then be solved if the series converges. Let $R = \max|\sigma(\vec{r})|$, then $|\sigma(\vec{K})| \le RA$, where A is the area covered by a data set. With $H = \max|h(\vec{r})|$, then

$$g(\vec{K}) = 2\pi G\sigma(\vec{K})\exp[-|\vec{K}|Z_0]\sum_{n=0}^{\infty}\frac{[-|\vec{K}|h(\vec{r})]^n}{n!}$$

$$\le 2\pi GAR\sum_{n=0}^{\infty}\frac{(H|\vec{K}|)^n}{n!}\exp(-|\vec{K}|Z_0) = 2\pi GAR\sum_{n=0}^{\infty}L_n\,,$$

where

$$L_n = \frac{\left(H|\vec{K}|\right)^n}{n!}\exp\left(-|\vec{K}|Z_0\right) = \frac{\left(H|\vec{K}|\right)^n / n!}{\sum_{j=0}^{\infty}\frac{\left(|\vec{K}|Z_0\right)^j}{j!}} < \left(\frac{H}{Z_0}\right)^n ,$$

independently of the value of $\vec{K}$. Because we can choose the plane E below the observation surface S $(Z_0 > H)$, the series in equation (4) is uniformly convergent over the entire wavenumber domain, by the Weierstrass M-test (Whittaker and Watson 1962, p. 49). This property is the same as the convergence of Parker's formula (Parker, 1973).

Based on Poisson's relation (Dobrin, 1976, p. 483), the formula for the magnetic anomaly can be written directly from equation (4),

$$T(\vec{K}) = 2\pi\frac{(\vec{K}\cdot\vec{f})(\vec{K}\cdot\vec{m})}{|\vec{K}|}J(\vec{K})\exp\left[-|\vec{K}|Z_0\right]\sum_{n=0}^{\infty}\frac{\left[-|\vec{K}|h(\vec{r})\right]^n}{n!} , \quad (5)$$

where " $\cdot$ " is the dot product of vectors,

$\vec{K} = i(k_x\vec{e}_x + k_y\vec{e}_y) + \sqrt{k_x^2 + k_y^2}\,\vec{e}_z$, $\vec{e}_z$ is the unit vector in the z direction, $\vec{f}$ and $\vec{m}$ are the unit vectors of the Earth's field and the magnetization, respectively.

Equations (4) and (5) allow us to estimate equivalent source $\sigma(\vec{r})$ (or $J(\vec{r})$) on the plane E from measured anomalies on the observation surface S by an iterative method, which is described below.

1) Initialize the equivalent source $\sigma(\vec{r})$ (or $J(\vec{r})$) and define the depth to $\sigma(\vec{r})$ (or $J(\vec{r})$);

2) Calculate the modeled gravity (or magnetic) anomaly from $\sigma(\vec{r})$ (or $J(\vec{r})$) by the formulas (4) (or (5));

3) Estimate errors: two errors used to control the iterative procedure are an rms error $RMS(k)$ at the kth iteration

$$RMS(k) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(s_i - u_i^k\right)^2} \quad (6)$$

and the maximum deviation $MAXD(k)$ at the kth iteration

$$MAXD(k) = \max_{1\le i\le N}\left|s_i - u_i^k\right| , \quad (7)$$

where superscript k stands for the kth iteration and subscript i for the ith data point; s is the measured anomaly; u is modeled anomaly calculated by formulas (4) (or (5)); and N is the total number of data points. If at an iteration, neither of these errors are reduced or the RMS reaches the accuracy threshold, the iterative procedure will be terminated. Otherwise:

4) Modify the $\sigma(\vec{r})$ (or $J(\vec{r})$) based on formulas below, then go to step 2);

$$\sigma_i^{k+1} = \sigma_i^k + \left(s_i - g_i^k\right)/2\pi G \quad \left(\text{or } J_i^{k+1} = J_i^k + C\left(s_i - T_i^k\right)\right), \quad (8)$$

where s is measured gravity (or magnetic) anomaly; g and T are calculated based on formulas (4) and (5), respectively; and C is a constant, which is chosen to make the iterations convergent. In our experience, C is chosen as 0.1 when J and T are in the same units. These simple formulas of modification are reliable and save time.

Once the equivalent source on the plane E is determined, the field on a horizontal plane (corrected datum) above the plane E is the normal upward continuation by formula (2). In this case, function $Z(\vec{r})$ is a constant, which is the vertical distance from the corrected datum to the plane E. The equivalent source can also be used to calculate pseudo-gravity, anomaly migrated to pole, directional directives, etc.

TESTING BY SYNTHETIC MODELS

The first example is from Xia and Sprowl (1991). A point-mass gravity source is buried 100 m directly beneath a 100 m vertical scarp at the surface. The source has an excess mass of 10^{10} kg. The data are a 15 × 15 grid of points, spaced every 100 m. Figures 2a and 2b plot Bouguer anomaly measured on the scarp and taken on a horizontal datum 200 m above the source (z = -100 m), after "normal" data corrections to a common datum have been performed. The distortion in the measured anomaly is due to the decreased vertical separation between the source body and the lower measuring stations. The initial equivalent density is set to 0 and the depth to equivalent density is 1m (z is positive downward). The initial RMS and $MAXD$ errors are 0.316 mGal and 2.358 mGals. After 11 iterations, the RMS and $MAXD$ errors between the modeled anomaly caused by the equivalent source and the anomaly on the scarp (Figure 2a) are reduced to 0.009 mGal and 0.126 mGal, respectively. We use the equivalent source to calculate the corrected anomaly on the datum z = -100 m, which is plotted in Figure 2c. The RMS error between the corrected anomaly (Figure 2c) and the true anomaly (Figure 2b) is 0.012 mGal. Figure 2d shows the difference between Figures 2b and 2c. The maximum and average values of correction (here we define the value of correction as the difference between the measured data and corrected data on a given datum. In this case, they are the difference between Figures 2a and 2c), for the example are 1.203 mGals and 0.059 mGal, respectively.

Our experience shows that the error caused by this type of equivalent source (continuously covered on a plane) is not sensitive to the depth of the equivalent source. To confirm this, we set the equivalent source at different depths from z = 0.001 m to 100 m for the example. The results show that the RMS error between the corrected and true anomalies on the datum z = -100 m is in the region 0.011- 0.012 mGal. However, the number of iterations increases from 7 to 53 with increasing the depth to equivalent source from 0.001m to 100m.

The second example is a rectangular solid buried beneath the scarp with inclination = 60°, declination = 30°, and magnetization = 4 A/m. The solid is 100 × 100 × 100 m in the center of the data area and has its top at 50 m depth and its bottom at 150 m depth. Figures 3a and 3b show the magnetic anomaly on the scarp and on the datum z = -50 m. The initial magnetization is set to 0 and the depth to equivalent magnetization is 1 m. The initial RMS and $MAXD$ errors are 11.618 nT and 76.580 nT, respectively. The RMS and $MAXD$ errors are reduced to 0.490 nT and 3.246 nT, respectively, after 42 iterations. Figure 3c shows the corrected anomaly on the datum z = -50 m. The RMS error and average deviation between the corrected anomaly (Figure 3c) and the true anomaly (Figure 3b) are 2.109 nT and 0.743 nT, respectively. If the corrected datum is chosen as z = -100 m, the RMS error and average deviation between the corrected anomaly and true anomaly will be 0.520 nT and 0.250 nT, respectively. Figure 3d shows the difference between Figures 3b and 3c. The maximum and average values of correction are 69.159 nT and 2.553 nT, respectively.

POTENTIAL-FIELD DATA IN KANSAS

Bouguer Gravity. There are more than 52,000 gravity stations measured on the topographic surface in Kansas. The highest point on the topography is 1,231.1 m above the sea level in western Kansas and the lowest is 215.2 m above the sea level in eastern Kansas. We used SURFACE III (Sampson, 1989) to grid Bouguer gravity data by the kriging method to 1.6 × 1.6 km. The final gridded data set is 205 × 408 points. The original Bouguer anomaly map is shown in Lam and Yarger (1989).

The initial equivalent density is 0 and the depth to the equivalent source is set to z = -200 m, just below the lowest point. The initial *RMS* and *MAXD* errors are 79.9 mGals and 151.1 mGals, respectively. The *RMS* and *MADX* errors are reduced to 0.1 mGal and 1.7 mGals, respectively, after 2 iterations. In each iteration, the value of the second term in the series (4) is about 5 percent of the first term, which means that the series (4) converges rapidly. The same thing happens in the magnetic case below. We use the equivalent density to calculate the corrected Bouguer anomaly on the datum z = -700 m, which is shown in Figure 4. The maximum and average values of correction are 2.55 mGals and 0.18 mGal, respectively. The calculations took about 4 minutes on a Data General MV 20000.

Aeromagnetic anomaly. There are about 72,000 line-km (8 - 11 data /km) of aeromagnetic data in Kansas. The distance between the flight line is 3.2 km. The data were measured on three different levels, 762.0 m above the sea level in eastern Kansas, 914.4 m and 1,371.6 m above the sea level in the east part and west part of western Kansas, respectively. There is a transition zone about 5 - 15 km wide in about the middle of western Kansas, in which the plane changed elevation from 914.4 m to 1,371.6 m. The elevations in the zone are linearly interpolated (Yarger, 1985 and 1989). We used SURFACE III (Sampson, 1989) to grid these data by the kriging method to 1.6 × 1.6 km. The final gridded data set is 205 × 408 points. Readers may refer to Yarger et al. (1981) for the original aeromagnetic map. The Kansas aeromagnetic map contains a constant shift betweem the eastern and western parts due to data acquisition factors. This correction constant was subtracted prior to equivalent source determination.

The initial equivalent magnetization is 0 and the inclination and declination are chosen as 65° and 7°, respectively. The depth to equivalent source is set to 760 m above the sea level (z = -760 m), just below the lowest level of the survey. The initial *RMS* and *MAXD* errors are 190 nT and 1,106 nT, respectively. The *RMS* and *MAXD* errors are reduced to 4 nT and 20 nT, respectively, after 12 iterations. The calculations took about 20 minutes on a Data General MV20000. We used the equivalent magnetization to calculate the corrected anomaly on three different levels, z = -762.0 m , -914.4 m, and -1,371.6 m. The results are shown in Table 1. When the corrected datum coincides with the one of measurement levels, the average value of the corrections is approximately the *RMS* error between the modeled anomaly from the equivalent source and measured anomaly, which means no correction preformed to the data in this case. Table 1 also shows that the amount of correction on the different levels is reasonable. Amount of correction increases with amount of vertical distance change. Figure 5 shows the corrected aeromagnetic anomaly on the datum z = -914.4 m.

ACKNOWLEDGEMENTS

The authors wish to thank Don Steeples and Rick Miller of Kansas Geological Survey for access to the Survey computing system and gravity data base. The authors also appreciate the efforts of Esther Price in manuscript preparation. One author (JX) thanks Dr. Steeples and Dr. Miller for the opportunity to study at the KGS, and the KGS for financial support.

REFERENCES

Bhattacharyya, B. K., and Chan, K. C., 1977, Reduction of magnetic and gravity data on an arbitrary surface acquired in a region of high topographic relief: Geophysics, **42**, 1411-1430.

Dampney, C. N. G., 1969, The equivalent source technique: Geophysics, **34**, 39-53.

Dobrin, M. B., 1976, Introduction to geophysical prospecting (third edition): McGraw-hill Book Company.

Gusip, F., 1987, Frequency-domain reduction of potential field measurements to a horizontal plane: Geoexpl., **24**, 87-98.

Henderson, R. G., and Cordell, L., 1971, Reduction of unevenly spaced potential field data to a horizontal plane by means of finite harmonic series: Geophysics, **36**, 856-866.

Lam,C. K., and Yarger, H. L., 1989, State gravity map of Kansas: Kansas Geological Survey, Bulletin **226**, 185-196.

Parker, R. L., 1973, The rapid calculation of potential anomalies: Geophys. J. Roy. Astr. Soc., **31**, 447-455.

Pilkington, M., and Urquhart., W. E. S., 1990, Reduction of potential field data to a horizontal plane: Geophysics, **55**, 549-555.

Pilkington, M., 1990, Private communication.

Sampson, R., 1989, SURFACE III: Interactive Concepts Inc.

Syberg, F. J. R., 1972, Potential field continuation between general surfaces: Geophys. Prosp., **20**, 267-282.

Tsuboi, C., 1965, Calculation of Bouguer anomalies with due regard to the anomaly in the vertical gravity gradient: Japan Acad. Proc., **41**, 386-391.

Whittaker, E. T., and Watson, G. N., 1962, A course of modern analysis: Cambridge University Press, Cambridge.

Xia, J., and Sprowl, D. R., 1991, Correction of topographic distortions in gravity data: Geophysics, **56**, 537-541.

Yarger, H. L., Robertson, R. R.,Martin, J. A., Ng, K., Sooby, R. L., and Wentland, R. L., 1981, Aeromagnetic map of Kansas: Kansas Geological Survey, Map *M*-16, scale 1:500,000, color interval 200 nT, contour interval 50 nT.

Yarger, H. L., 1985, Kansas basement study using spectrally filtered aeromagnetic data: *in*, The Utility of Regional Gravity and Magnetic Anomaly Map, W.J.Hinze, *ed.*: Society of Exploration Geophysics; Kansas Geological Survey , Open-file Report **83-9**, 36 p. (1983).

Yarger, H. L., 1989, Major magnetic features in Kansas and their possible geologic significance: Kansas Geological Survey, Bulletin **226**, 197-213.

Table 1. Values of correction of the aeromagnetic data on three parts of Kansas.

Datum (m)	Average values of correction (nT): Western Kansas			Maximum correction (nT)
	West part	East part	Eastern Kansas	
-762.0	14	5	4	383
-914.4	11	3	5	270
-1371.6	3	10	14	270

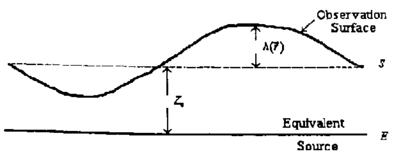

Fig. 1. Geometry of the plane of the equivalent source E and a generic observation surface S.

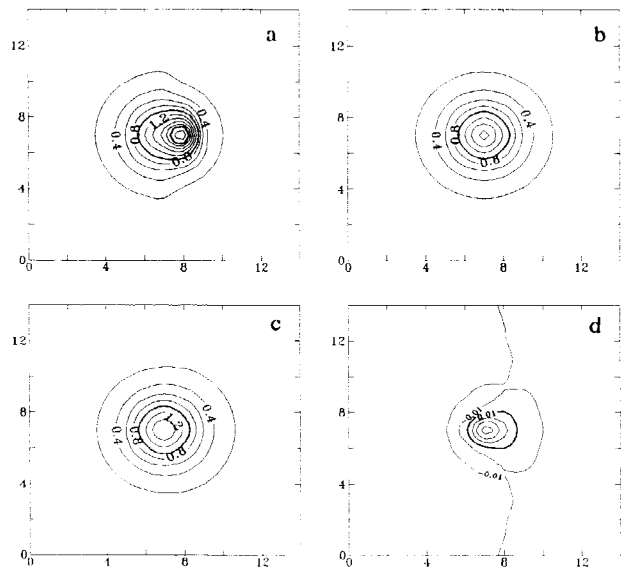

Fig. 2. The point-mass example (Xia and Sprowl, 1991). (a) is the Bouguer anomaly on the scarp and (b) is on the horizontal datum $z = -100$ m. (c) is the topographically corrected anomaly on the datum $z = -100$ m. (d) is the Bouguer anomaly the difference between the (b) and (c). The unit in both x and y directions is one station spacing, 100 m. Contour interval 0.2 mGal in (a) - (c), and 0.02 mGal in (d).

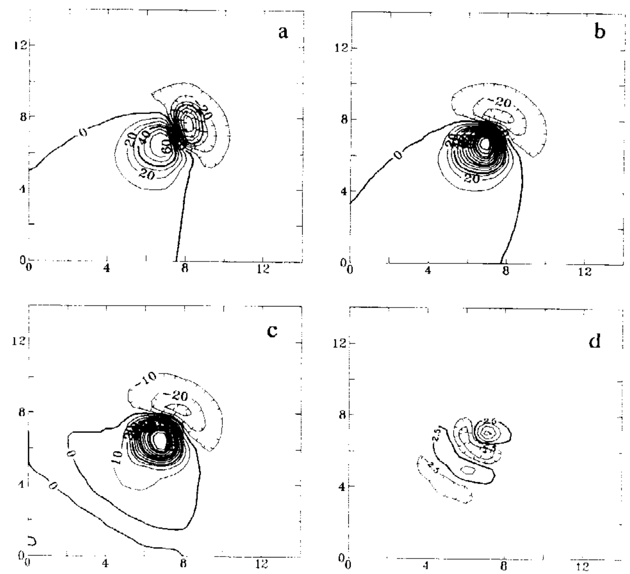

Fig. 3. The rectangular solid example. (a) is the magnetic anomaly on the scarp and (b) is the magnetic anomaly on the horizontal datum $z = -50$ m. (c) is the topographically corrected anomaly on the datum $z = -50$ m. (d) is the difference between the (b) and (c). The unit in both x and y directions is one station spacing, 100 m. Contour interval 10 nT in (a) - (c), 5 nT in (d).

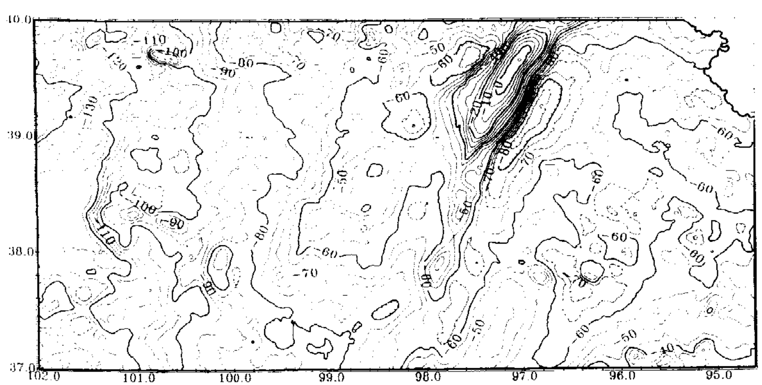

Fig. 4. Topographically corrected Bouguer gravity of Kansas on the datum 700 m above sea level. Coordinates in x and y directions are longitude and latitude in degrees. Contour interval is 10 mGals.

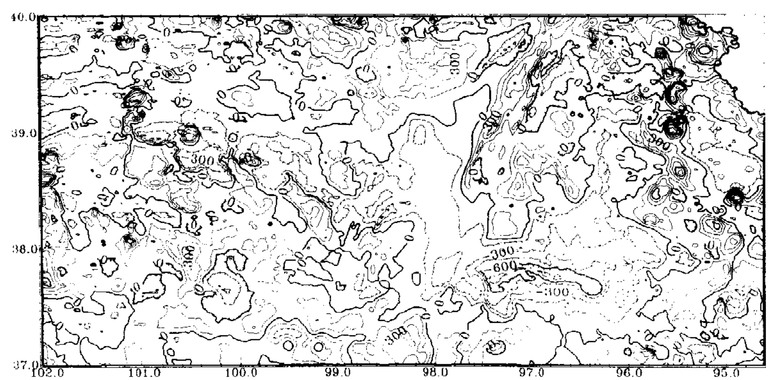

Fig. 5. Aeromagnetic anomaly of Kansas on the datum 914.4 m above sea level. Coordinates in x and y directions are longitude and latitude in degrees. Contour interval is 150 nT.