Inversion of Potential-Field Data by Iterative Forward Modeling in the Wavenumber Domain

GM3.2

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SUMMARY

Iterative forward modeling has been used to determine inversion of the gravity (or magnetic) data in the wavenumber domain. Modeled anomalies are calculated by Parker's (1973) formula. Two forms of inverse modeling are determined: density (magnetization) distribution in a specified layer, and interface with a constant density (magnetization) contrast. Because of the stability of the forward calculation, no particular filter or initial model are needed to produce convergence of the solution. The technique allows large data sets to be inverted.

A density interface derived from the inversion of gravity data (27,132 points) in the region of the Midcontinent Rift System (MRS) in Kansas produces a modeled gravity which agrees with observations to an RMS error of about 1% of the maximum anomaly value. The model fits drill control from the Texaco Poersch #1 well (Berendsen et al., 1988). The subsurface density model supports the interpretation that the MRS does not terminate in central Kansas, but continues along a south-west trend to at least the Kansas-Oklahoma border (Yarger, 1983).

The formula for calculating the gravity anomaly caused by a density interface with an exponential model has been derived.

INTRODUCTION

Inversion of geophysical data in the wavenumber domain has been of interest since Parker's formula was published. As discussed by Oldenburg (1974), a major problem is the instability of the technique because explicit or implicit downward continuation is involved. Therefore, more recent implementations have involved theoretical or empirical regularization filters to taper growth of the exponential continuation function (Parker and Huestis, 1974; Oldenburg, 1974; Granser, 1986; Ferguson et al., 1988; and Reamer and Ferguson, 1989). A direct inversion formula was derived by Grasner (1987). The convergence of this formula is restricted to low frequencies requiring filtering which causes loss of high-frequency information about the interface.

In the space domain, the method of Cordell and Henderson (1968) is commonly used. The density interface is divided into a number of vertical prisms by some averaging or interpolation scheme, and a linear function is applied to modify the depth of each prism after each iteration. This method takes longer than methods based on Parker's formula, especially for large data sets. The singular-value-decomposition technique can be used to determine a 2-D density interface (Xia, 1986), but is difficult to apply in three dimensions because there are too many unknowns.

In our study, inversion is accomplished through iterative improvement of an initial subsurface model. Modeled anomalies are calculated by using Parker's formula.

INVERSION APPROACH

Calculation of an anomaly field due to a material layer using Parker's formula requires three known functions: the depth to the top of the layer (ZT), the depth to the bottom of the layer (ZB) and the density (magnetization). For magnetic anomalies, the direction of magnetization must be known. The forward series expansion of Parker's formula is uniformly convergent for any reasonable topographic relief functions, ZT and ZB (Parker, 1973).

In our study, if ZB is assumed to be a constant ($ZB \geq ZT$), which implies that the anomaly is caused only by the upper interface ZT, the goal of inversion is to determine ZT under the condition of a known constant density (magnetization). Conversely, given ZT and ZB, the goal of inversion is to determine distribution function of density (magnetization) in the layer defined by ZT and ZB. The formulas that are used to modify the model after each iteration are listed below.

Case 1. Determining the depth to the top of magnetic interface:

$$\Delta H_i^k = 2\left[\tan(h_i^k / 4\bar{J}) - \tan(T_i / 4\bar{J})\right](ZT_i^k)^2 / \Delta L, \qquad (1)$$

where superscript k stands for the kth iteration and subscript i for the ith data point; ΔH_i^k, is the modification to the depth ZT_i^k, the depth of ZT below point i; h_i^k and T_i are the calculated and measured total magnetic field anomalies, respectively; and ΔL is the average distance between data points;

$$\bar{J} = J \times (\sin^2 I - \cos^2 I \times \sin^2 D),$$

where I and D are the angles of inclination and declination of magnetization J, respectively.

Case 2. Determining the distribution of magnetization in the layer:.

$$\Delta J_i^k = (T_i - h_i^k) / 4\tan^{-1}(\Delta L / 2ZT_i^k), \qquad (2)$$

where ΔJ_i^k is the modification to J_i^k, the magnetization below point i.

Case 3. Determining the depth to the top of density interface:

$$\Delta H_i^k = (g_i - h_i^k) / 2\pi G \Delta\rho, \qquad (3)$$

where g_i and h_i^k are the measured and calculated gravity anomalies, respectively, G is the gravitational constant, and $\Delta\rho$ is the density contrast.

Case 4. Determining the distribution of density in the layer:

$$\Delta\rho_i^k = (g_i - h_i^k) / 2\pi G(ZB_i - ZT_i), \quad (4)$$

the formulas (1) and (2) are simplified from the 2-D vertical dike (Telford, Geldart, Sherriff, and Keys, 1982, p.166). Formulas (3) and (4) are based on the Bouguer-slab formula.

Two errors used to trace the iterative procedure are an rms error *RMS(k)* at the *k*th iteration

$$RMS(k) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(h_i^k - f_i)^2}, \quad (5)$$

and the maximum deviation *MAXD(k)* at the kth iteration

$$MAXD(k) = \max_{1\le i\le N}\left|h_i^k - f_i\right|, \quad (6)$$

where f_i is measured anomaly, N total number of data points.

The inversion approach can be described as follows: 1) Determine an initial model: initialize the model *ZT* to a average depth of the interface and define $\Delta\rho$ (or *J*) = constant and *ZB* = constant (≥*ZT*) for the case 3 (or case 1), or initialize the model $\Delta\rho$ or *J*) to a average value and define *ZT* and *ZB* for the case 4 (or case 2); 2) Calculate anomaly *h(x,y)* by Parker's formula and the *RMS* and *MAXD* by formulas (5) and (6), if neither of these errors are reduced or the *RMS* reaches the accuracy threshold, the iterative procedure will be terminated, otherwise; 3) Modify the model by one of formulas (1) - (4) according to the goal of inversion and the type of anomaly, go to step 2.

SYNTHETIC EXAMPLE

We calculated the synthetic magnetic anomaly from a rectangular solid (I = 60°, D = 45°, magnetization = 400 nT). The solid is 80x80 km horizontally and has its top at 3 km depth and its bottom at 6 km depth; i.e., a square horst with 3 km vertical offset. Anomaly values were calculated at z = 0 on a 100x100 grid of points, spaced every 1.6 km.

Case 1. Define J = 400 nT and *ZB* = 100 km, initialize the model *ZT* = 4.5 km. Initial errors are *RMS* = 132.1 nT and *MAXD* = 626.3 nT. After 17 iterations, the errors are *RMS* = 37.2 nT (6% of the maximum anomaly) and *MAXD* = 128.9 nT. The final model, representing the modified interface *ZT*, is shown in Figure 1.

Case 2. Define *ZT* = 3 km and *ZB* = 6 km, initialize the magnetization to 200 nT The initial errors are the same as in case 1. After 20 iterations, the errors *RMS* = 28.9 nT (4.4% of the maximum anomaly) and *MAXD* = 99.2 nT. The final model, which represent the modified magnetization distribution in the layer, is shown in Figure 2. The sharp boundaries in Figures 1 and 2 clearly locate the position of the solid.

GEOLOGIC EXAMPLE

We invert gravity data from central Kansas in the region of the MRS. The data are the residual Bouguer anomaly with a second order trend removed (Lam and Yarger, 1989), gridded to 1.6x1.6 km by SURFACE III (Sampson, 1988). Total number of data points is 27,132. The initial model is based on well data (Berendsen et al., 1988) and models by Yarger (1989) and Somanas et al. (1989).

Case 3. Define $\Delta\rho$ = 0.5 g/cc, *ZB* = 100 km and initialize *ZT* to 2.6 km. The initial errors are *RMS* = 13.1 mgals and *MAXD* = 52.2 mgals. Two iterations reduce the errors to *RMS* = 0.86 mgals and *MAXD* = 6.8 mgals. The final model, which represents the modified depth to the top basaltic rocks, is shown in Figure 3. The calculations took 125 CPU seconds on a Data General MV20000. The depth to the top of basaltic rocks in Texaco Poersch #1 is about 0.9 km. The modeled interface has a depth 1.1 km at the same location.

Case 4. Define *ZT* = 0.8 km and *ZB* = 3.5 km, and initialize the density of the layer to 2.67 g/cc. The initial errors are the same as the case 3 above. After 2 iterations the errors are *RMS* = 0.5 mgal, and *MAXD* = 3.1 mgals. The final model of density distribution in the specified layer is shown in Figure 4. The southern extent of the rift system (presence of basaltic rocks) is suggested by 2500 m contour in Figure 3 and by the zone of densities greater than 2.7 g/cc in Figure 4. Rift associated volcanics appear to extend almost to the Kansas-Oklahoma border.

EXPONENTIAL DENSITY MODEL

In the future basin modeling, a density which is an exponential function of depth

$$\rho(z) = a + be^{-\mu z} \quad (7)$$

will be assumed. The gravity anomaly caused by the density interface with this density model is $h = h_1 + h_2$. The first term h_1, which is caused by a constant density a, can be calculated by Parker's formula. The formula of calculating the gravity anomaly h_2 caused by the second exponential term has been derived.

$$F[h_2(x,y)] = 2\pi Gb\sum_{n=1}^{\infty}\frac{\left[-(\mu+|\vec{K}|)\right]^{n-1}}{n!}\times$$
$$\left\{e^{-(\mu+|\vec{K}|)\delta_2}F\left[(z_2(x,y)-\delta_2)^n\right]-\right.$$
$$\left.e^{-(\mu+|\vec{K}|)\delta_1}F\left[(z_1(x,y)-\delta_1)^n\right]\right\} \quad (8)$$

the notations of this formula are the same as in Blakely (1981). It is apparent that calculation of h_2 will require the same order of magnitude of time as the calculation of h_1 The linear density model of Reamer and Ferguson (1989) is the specific case of the exponential model (7). Formula (8) may be more computational efficiency than the formula which is based on using vertical prisms with the model (7) to fit an interface (Chai and Hinze, 1988; Chenot and Debeglia, 1990). Further investigation is warranted.

CONCLUSIONS

The forward, iterative approach employed here avoids the problem of growth of the exponential continuation function and is efficient enough to be applied to large data sets. The solution converges stably and is reliable except for the case of inversion of magnetic data with inclination less than 45° (migration-to-pole before inversion should help in such cases). This approach allows the model of density (magnetization) distribution in a flexible layer, a specific case of which is a horizontal layer. This flexibility is useful for determining the density (magnetization) distribution in a particular layer. The distribution of density (magnetization) could be more closely analogous to a geologic map (Cordell and McCafferty, 1989) and probably provide the information about the boundaries of different types of rocks or geological structures.

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REFERENCES

Berendsen, P., Borcherding, R.M., Doveton, J., Gerhard, L., Newell, K.D., Steeples, D., and Watney, W.L., 1988, Texaco Poersch #1, Washington County, Kansas-Preliminary geologic report of the pre-Phanerozoic rocks: Kans. Geol. Survey, Open-file report 88-22.

Blakely, R.J., 1981, A program for rapidly computing the magnetic anomaly over digital topography: U.S. Geol. Survey, Open-file report 81-298.

Chenot, D., and Debeglia, N., 1990, Three-dimensional gravity or magnetic constrained depth inversion with lateral and vertical variation of contrast: Geophysics, 55, 327-335.

Chai, Y., and Hinze, W.J., 1988, Gravity inversion of an interface above which the density contrast varies exponentially with depth: Geophysics, 53, 837-845.

Cordell, L., and Henderson, R.B., 1968, Iterative three-dimensional solution of gravity anomaly data using a digital computer: Geophysics, 33, 596-601.

Cordell, L., and McCafferty, A.E., 1989, A terracing operator for physical property mapping with potential field data: Geophysics, 54, 621-634.

Granser, H., 1986, Convergence of iterative gravity inversion: Geophysics, 51, 1146-1147.

Granser, H., 1987, Nonlinear inversion of gravity data using the Schmidt-Lichtenstein approach: Geophysics, 52, 88-93.

Ferguson, J.F., Felch, R.N., Aiken, C.L.V., Oldow, J.S., and Dockery, H., 1988, Model of the Bouguer gravity and geologic structure at Yucca Flat, Nevada: Geophysics, 53, 231-244.

Lam. C., and Yarger, H., 1989, State gravity map of Kansas: Kansas Geol. Survey Bull. 226, 185-196.

Oldenburg, D.W., 1974, The inversion and interpretation of gravity anomalies: Geophysics, 39, 526-536.

Parker, R.L., 1973, The rapid calculation of potential anomalies: Geophys. J. Roy. Astr. Soc., 31, 447-455.

Parker, R.L., and Huestis, S.P., 1974, The inversion of magnetic anomalies in the presence of topography: J. Geophys. Res., 79, 1587-1593.

Reamer, S.K., and Ferguson, J.F., 1989, Regularized two-dimensional Fourier gravity inversion method with application to Silent Canyon caldera, Nevada: Geophysics, 54, 486-496.

Sampson, R., 1988, Surface III: Interactive Concepts Inc., 277 p.

Somanas, C., Knapp, R.W., Yarger, H., and Steeples, D., 1989, Geophysical model of the Midcontinent Geophysical Anomaly in northeastern Kansas: Kansas Geol. Survey Bull. 226, 215-228.

Telford, W.M., Geldart, L.P., Sheriff, R.E., and Keys, D.A., 1982, Applied geophysics: Cambridge University Press, 860 p.

Yarger, H., 1983, Regional interpretation of Kansas aeromagnetic data: Kansas Geol. Survey, Geophysics series 1, 35 p.

Yarger, H., 1989, Major magnetic features in Kansas and their possible geologic significance: Kansas Geol. Survey Bull. 226, 197-207.

Xia, J., 1986, Solving inverse problem of two-dimensional single density interface by singular value decomposition: Computing Techniques for Geophysical and Geochemical Exploration, Ministry of Geology and Minerals, China, 8, 128-134.

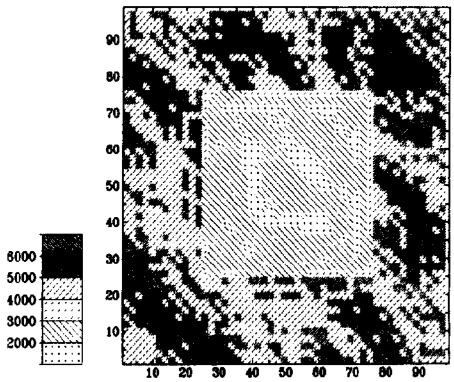


FIG. 1. Depth to magnetic interface. Units in meters.

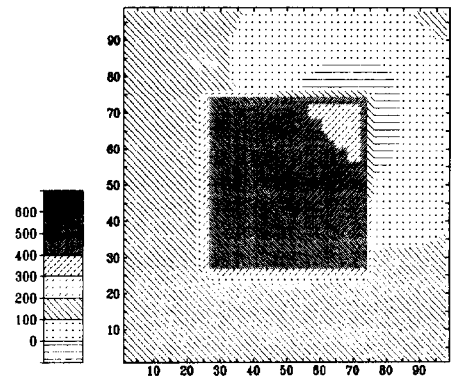
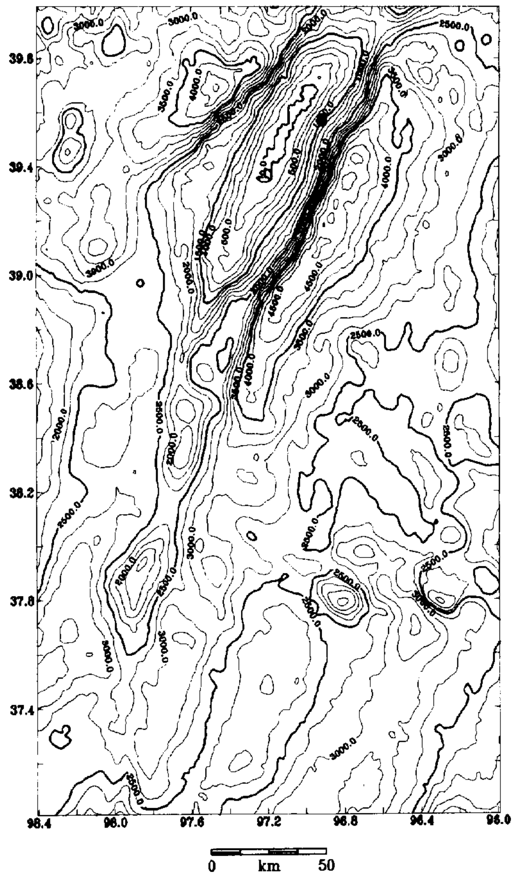


FIG. 2. Distribution of magnetization within a 3 km thick slab lying between 3 km to 6 km deep. Units in nTs.



● The location of Texaco Poersch #1.

FIG. 3. Depth to density interface of MRS, Kansas. Datum is the same as the elevation of Texaco Poersch #1. Contour interval is 250 m.

FIG. 4. Density distribution in MRS area, Kansas, within a 2.7 km thick slab lying between 0.8 km to 3.5 km depth. Units in g/cm3.